ELSEVIER Journal of Economic Dynamics & Control 26 (2002) 1397–1416

JOURNAL OF
Economic
Dynamics
& Control

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An anticipative feedback solution for the infinite-horizon, linear-quadratic, dynamic, Stackelberg game

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Received 17 September 2001

Abstract

This paper derives and illustrates a new suboptimal-consistent feedback solution for an infinite-horizon, linear-quadratic, dynamic, Stackelberg game. This solution lies in the same solution space as the infinite-horizon, dynamic-programming, feedback solution but puts the leader in a preferred equilibrium position. The idea comes from Kydland (J. Econ. Theory 15 (1977)) who suggested deriving a consistent feedback solution for an infinite-horizon, linear-quadratic, dynamic, Stackelberg game by varying the coefficients in the player's linear constant-coefficient decision rules. Here feedback is understood in the sense of setting a current control vector as a function of a predetermined state vector. The proposed solution is derived for discrete- and continuous-time games and is called the anticipative feedback solution. The solution is illustrated with a numerical example of a duopoly model. © 2002 Elsevier Science B.V. All rights reserved.

JEL classification: C61; C63; C73

Keywords: Noncooperative games; Solving Riccati-type nonlinear algebraic equations

1. Introduction

Dynamic Stackelberg (or leader-follower) games are useful tools for studying dynamic economic behavior in equilibrium settings in which some player is dominant. Because of their tractability, infinite-horizon, linear-quadratic, dynamic, Stackelberg (LQDS) games have received particular attention. LQDS games have been used to

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PII: S0165-1889(01)00077-X

study noncompetitive behavior in specific markets and to evaluate and design macroeconomic policies. For example, Sargent (1985) contains studies of energy markets based on LQDS games; Kydland and Prescott (1977) and Fischer (1980) studied optimal tax policy using DS games; Canzoneri and Gray (1985), Miller and Salmon (1985), and Turnovsky et al. (1988) studied international macroeconomic policy coordination using DS games. Section 4 illustrates the present anticipative feedback solution in a LQDS game of a hypothetical industry. The anticipative feedback solution could be applied to the LQ approximation of any dynamic economic setting with a dominant agent.

Three decision spaces have been considered in dynamic games: open-loop, feedback, and closed-loop. In open-loop decisions, players set their control vectors as functions of time; in feedback decisions, players set their control vectors as functions of the current (or most recently determined or observed) state vector; and in closed-loop decisions, players set their control vectors as functions of the history of the state vector, from the start of the game to the moment of decision. For example, Hansen et al. (1985) considered open-loop solutions of discrete-time LQDS games, computed using Euler-type equations. Simaan and Cruz (1973) considered feedback solutions of DS games, computed using backwards recursions of dynamic programming. To emphasize the dynamic programming nature of these feedback solutions, we refer to them as dynamic programming feedback (DPF) solutions. Basar and Selbuz (1979), Basar and Olsder (1980), and Tolwinski (1981) considered classes of closed-loop solutions for discrete- and continuous-time, DS games, computed using nonstandard (nonDP) recursions and differential equations. See Basar and Olsder (1995, Chapter 7) for a comprehensive discussion of DS games.

A potential problem in DS games is that the solution which is optimal for the leader at the beginning of the game is time inconsistent. That is, it ceases to be optimal for the leader in subsequent periods. Consequently, the leader has an incentive to restart the game. In a rational-expectations setting, followers would recognize continual restarts. Such a succession of restarted leader-optimal solutions would be unsustainable and, hence, unappealing as a solution concept. The time inconsistency problem in DS games was first noted by Simaan and Cruz (1973), Kydland (1975, 1977), and Kydland and Prescott (1977) for open-loop solutions of DS games. In response, Simaan and Cruz (1973), Kydland (1975, 1977), and Kydland and Prescott (1977) considered DPF solutions of DS games. DPF solutions are time consistent by construction, but do not entirely solve the time consistency problem because in them the leader is continually tempted to switch to an optimal, open- or closed-loop, solution.

Basar and Selbuz (1979) and Basar and Olsder (1980) proposed closed-loop solutions for discrete- and continuous-time, DS games. The Basar–Selbuz–Olsder solutions require additional structural restrictions, beyond the usual concavity (or convexity), playability, and stability conditions (see Section 2). However, whenever they are applicable, the Basar–Selbuz–Olsder solutions are time consistent. Nevertheless, even when applicable, the Basar–Selbuz–Olsder solutions are not subgame perfect. Tolwinski (1981) proposed a more general closed-loop solution for LQDS games (under weaker structural restrictions) which is nearly subgame perfect: if the follower deviates from the optimal solution path for some reason, the leader induces them