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Journal of Economic Theory 120 (2005) 73–107
JOURNAL OF Economic Theory
http://www.elsevier.com/locate/jet

On the endogeneity of Cournot–Nash and Stackelberg equilibria: games of accumulation

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Received 11 October 2002; final version received 10 December 2003

Abstract

We characterize equilibria of games with two properties: (i) Agents have the opportunity to adjust their strategic variable after their initial choices and before payoffs occur; but (ii) they can only add to their initial amounts. The equilibrium set consists of just the Cournot–Nash outcome, one or both Stackelberg outcomes, or a continuum of points including the Cournot–Nash outcome and one or both Stackelberg outcomes. A simple theorem that uses agents' standard one-period reaction functions and the one-period Cournot–Nash and Stackelberg equilibria delineates the equilibrium set. Applications include contribution, oligopoly, and rent-seeking games.

JEL classification: C72; C73; L13; H41

Keywords: Accumulation games; Cournot–Nash outcome; Stackelberg outcome

1. Introduction

In a variety of settings, agents repeatedly interact and take irreversible actions before payoffs accrue. For instance, donors can make multiple non-refundable contributions to a public good, lobbies repeatedly engage in rent-seeking activities to influence a policy decision,[1] and duopolists can add to their previous stock of output

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[1] For instance, in the U.S., interest groups in different industries can repeatedly make campaign contributions to congressmen before a policy decision will be made at a predetermined date. Very soon after contributions are made, their amounts, the recipient congressmen, and the timing of contributions become public information. This information is posted at www.opensecrets.org.

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doi:10.1016/j.jet.2003.12.005

before the market clears. Such games have two main features: (i) Before payoffs occur, agents have multiple opportunities to vary their strategic variable and to observe their opponent's most recent strategy choice; but (ii) they can only accumulate their strategic variable over time.

With some exceptions discussed below, studies of such games have assumed that agents make their choices once and that they interact either in a standard Cournot–Nash or Stackelberg fashion. While these modeling approaches provide valuable insights into the nature of agents' choices and the equilibrium outcomes, a more realistic specification of such games should embody the two elements discussed above. Our objectives in this paper are to determine the consequences of the possibility of "strategic accumulation" for a large set of games, and to examine the implications in a variety of applications. The contribution of this paper is twofold: On the technical side, we are able to solve this set of games in a unified manner thus allowing us to highlight the common themes; and, on the application side, we show how some predictions of previously analyzed models might change dramatically once we account for the dynamics and irreversibility of initial actions. As a byproduct, our study also allows us to identify the environments where leadership roles arise endogenously.

A brief preview of our main findings and the organization of our paper are as follows. We present the model in Section 2. Two agents are present whose preferences and strategy spaces are common knowledge. Agents simultaneously make initial choices, and, after these are observed, simultaneously choose whether to increase their strategic variable. Payoffs depend on the accumulated values.

In Section 3, we characterize the equilibrium set. We show that the equilibrium set can be delineated using the standard one-period reaction functions and the standard Cournot–Nash and Stackelberg outcomes. This characterization provides a convenient program for identifying the equilibrium possibilities in different scenarios.

Next we focus on when the Cournot–Nash outcome is the unique equilibrium. *The* necessary and sufficient condition is simply that each agent's standard Stackelberg-leader choice is less than his Cournot–Nash amount. This finding provides insight into the nature of the accumulation game. An example with this outcome is the standard model of private contributions to a public good where agents would like to free ride. A standard Stackelberg leader would free ride by committing to a low contribution—below the Cournot–Nash amount—knowing that this would induce a relatively high contribution by the follower. If, however, the Stackelberg leader could contribute again along with the "follower," then the leader's incentive to do so would lead back to the Cournot–Nash outcome. This intuition holds generally in this case thereby ruling out all but the Cournot–Nash outcome.

In other settings, equilibria in the accumulation game are equivalent to one or both of the standard Stackelberg outcomes. An example is duopoly quantity competition by producers of complements. Here initial choice of the standard Stackelberg leader's amount constitutes a credible commitment to maintaining that output because it exceeds the Cournot–Nash quantity (and this initial choice is an equilibrium strategy). The other possibility is to have a continuum of equilibria