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On Stackelberg competition in strategic multilateral exchange

Ludovic A. Julien *

LEG, Université de Bourgogne, EconomiX, Université Paris Ouest La Défense, France, and IRES, Université Catholique de Louvain, Belgique

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ABSTRACT

In this paper, we consider a strategic equilibrium concept which extends Stackelberg competition to cover a general equilibrium framework. From the benchmark of strategic market games proposed by Sahi and Yao (1989), we define the notion of Stackelberg equilibrium. This concept captures strategic interactions in interrelated markets on which a finite number of leaders and followers compete on quantities. Within the framework of an example, convergence and welfare are studied. More specifically, we analyze convergence toward the competitive equilibrium and make welfare comparisons with other strategic equilibria.

1. Introduction

Strategic interactions in non-cooperative general equilibrium are essentially based on Cournotian behaviors (Gabszewicz and Vial, 1972, Codognato and Gabszewicz, 1991, 1993; Codognato and Ghosal, 2000; Busetto et al., 2008, 2011). In this paper, we introduce a strategic general equilibrium concept, called Stackelberg equilibrium (SE hereafter). This concept generalizes the Stackelberg–Walras equilibrium (SWE) and the Stackelberg–Cournot equilibrium (SCE) concepts developed in Julien and Tricou (2010). The SWE considered one leader and several followers facing price-taking traders, while the SCE envisaged one leader competing with several followers. The SE involves several leaders and followers interacting strategically on all markets. Therefore, no competitive behavior is assumed. One objective is to provide a definition for the SE in a finite exchange economy. In addition, the consequences of several kinds of strategic behaviors are featured within an example in which convergence and welfare are studied.

General oligopoly equilibrium theory has been developed in two main directions. The first is the Cournot–Walras equilibrium (CWE) models proposed by Gabszewicz and Vial (1972) in an economy with production, and pursued in pure exchange by Codognato and Gabszewicz (1991, 1993) and Gabszewicz and Michel (1997). This class of models includes agents who behave strategically and others who behave competitively. These models are initially based on a Walrasian market price mechanism and aim at studying the consequences of market power within interrelated markets. The second line is the Cournot equilibrium (CE) developed in strategic market games (Shapley and Shubik, 1977; Dubey and Shubik, 1978; Sahi and Yao, 1989; Amir et al., 1990, among others). These models aim at circumventing the auctioneer. In this paper, we incorporate Stackelberg competition in the strategic market games model of Sahi and Yao (1989). Two reasons explain this. First, this class of models generate consistent market prices. Second, Stackelberg competition assumes that all traders behave strategically. So, we focus only on strategic behaviors.

The problems of existence and uniqueness – which raise specific difficulties in the context of a general oligopoly equilibrium – are beyond the scope of the paper. We also consider the position of traders in the timing of decision as given,

* Tel.: +33 1 4097 7543; fax: +33 1 4097 5907.
E-mail address: ludovic.julien@u-paris10.fr

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and therefore do not question how a trader could or should become a leader. We here rather define an equilibrium concept, and merely consider its link with related concepts in the literature. Three interesting features can be put forward when incorporating Stackelberg (1934) competition into a general equilibrium environment. First, the analysis of strategic interactions which recover from sequential decisions is complexified in interrelated markets. Hence, the strategic general equilibrium concepts are associated with different kinds of strategic behaviors. In addition, the consequences of imperfectly competitive behaviors depend on the fundamentals (preferences and endowments). Third, there is no exogenous pricing rule derived from a given market demand function: consistent market prices are here determined endogenously.

Within the framework of an exchange economy, we characterize and define the SE. Then, we provide an example into which convergence and welfare are studied. Thus, it is shown that, by replicating the economy an infinite number of times, the SE converges to the competitive equilibrium (WE) when the number of leaders increases unboundedly. The same result holds for the SCE and the SWE. These equivalences are specific to a general equilibrium perspective. In addition, it is shown that the SE and the SCE are Pareto dominated by the WE, while the SWE is not. Finally, we compare the Stackelberg equilibria with other strategic general equilibrium concepts. Hence, we show that the SWE is not Pareto dominated by the CWE, but that the CE Pareto dominates the SE. These results extend usual results in partial equilibrium analysis (first and second parts of Result 2, and Result 3), while others only hold in a general equilibrium framework (third part of Result 2 and Result 4).

The paper is organized as follows. Section 2 presents the framework and gives a definition for the SE. Section 3 provides an example. Section 4 is devoted to convergence. Section 5 deals with welfare. In Section 6, we conclude.

2. The Stackelberg equilibrium

Consider a pure exchange economy $\mathbf{E}$. The space of commodities is $\mathbb{R}^l_+$. The economy thus includes a finite set $\mathcal{L}$ of divisible consumption goods, indexed by $h=1,2,\ldots,l$. The space of traders is $\mathcal{I}$: it embodies a finite set of n traders, each being indexed by i, $i=1,\ldots,n$. Let us denote by $\mathcal{I}_h$ the subset of traders who are endowed with h, $h\in\mathcal{L}$, so $\mathcal{I}=\cup_{h=1}^{l}\mathcal{I}_h$. There are leaders and followers endowed with h, $h\in\mathcal{L}$. Thus $\mathcal{I}_h=\mathcal{I}_h^L\cup\mathcal{I}_h^F$, where $\mathcal{I}_h^L$ and $\mathcal{I}_h^F$ represent the subsets of leaders and followers endowed with $h\in\mathcal{L}$. Any trader $i\in\mathcal{I}$ is represented by his vector of endowments $\boldsymbol{\omega}^i$, his utility function $U^i(\cdot)$ which represents his preferences among the commodity bundles $\mathbf{x}^i\in X^i$ (his consumption set), and his strategy set B^i. Three assumptions are made regarding $\boldsymbol{\omega}^i$, $U^i(\cdot)$ and B^i.

Assumption 1. For all $i\in\mathcal{I}$, $U^i: X^i\subseteq\mathbb{R}^l_+\to\mathbb{R}$, $\mathbf{x}^i\mapsto U^i(\mathbf{x}^i)$ is continuous, strictly increasing and strictly quasi-concave on $\mathbb{R}^l_{++}$.

Assumption 2. The distribution of endowments among traders satisfies:

$$\begin{aligned}&(\mathbf{a})\ \text{For all } i\in\mathcal{I},\quad \text{there is one and only one } h:\omega_h^i>0,\\ &(\mathbf{b})\ \text{For all } h\in\mathcal{L},\quad \mathcal{I}_h\neq\{\emptyset\}.\end{aligned} \tag{1}$$

From **A2a**, we consider to simplify corner endowments. **A2b** says that any commodity is held, and thereby traded. Trader $i\in\mathcal{I}$ uses $\boldsymbol{\omega}^i$ to reach his allocation $\mathbf{x}^i\in\mathbb{R}^l_+$. A feasible allocation is an assignment $\mathbf{x}=(\mathbf{x}^1,\ldots,\mathbf{x}^i,\ldots,\mathbf{x}^n)$ for which $\sum_{i\in\mathcal{I}}\mathbf{x}^i=\sum_{i\in\mathcal{I}}\boldsymbol{\omega}^i$. The price vector is $\mathbf{p}\in\mathbb{R}^l_+$, with component p_h, $h\in\mathcal{L}$. **A1** and **A2** together guarantee the existence of a WE for $\mathbf{E}$.

Consider now the strategic market game $\boldsymbol{\Gamma}$ associated with $\mathbf{E}$. The strategic behavior involves all units of the owned good engaged in exchange.

Assumption 3. The strategy set B^i of trader i endowed with h satisfies:

$$B^i=\left\{b_{hk}^i\geq 0, k=1,\ldots,l:\sum_{k\in\mathcal{L}}b_{hk}^i\leq\omega_h^i\quad\text{and}\quad i\in\mathcal{I}_h\right\}, \tag{2}$$

where the bid b_{hk}^i indicates the amount of commodity h trader $i\in\mathcal{I}_h$ offers in exchange for commodity k, $k\in\mathcal{L}$. Therefore, a pure strategy for a trader $i\in\mathcal{I}_h$ is a vector $\mathbf{b}_h^i$, with component b_{hk}^i, $k=1,\ldots,l$.

A SE can be modeled as a two-step game, which is played under complete information. Each trader $i\in\mathcal{I}_h$ selects a strategy $\mathbf{b}_h^i$ in B^i. Given a strategy profile specified by $\boldsymbol{\Lambda}=(\mathbf{b}_1^1,\ldots,\mathbf{b}_\ell^i,\ldots,\mathbf{b}_l^n)$, where $\boldsymbol{\Lambda}$ is the matrix of bids formed by all vectors of strategies, the price system $\mathbf{p}(\boldsymbol{\Lambda})$ is the solution to

$$\sum_{h=1}^{l}p_h\left(\sum_{i\in\mathcal{I}_h^L}b_{hk}^i+\sum_{i\in\mathcal{I}_h^F}b_{hk}^i\right)=p_k\sum_{h=1}^{l}\left(\sum_{i\in\mathcal{I}_h^L}b_{kh}^i+\sum_{i\in\mathcal{I}_h^F}b_{kh}^i\right),\quad k\in\mathcal{L}. \tag{3}$$

Total supply to buy one good equals total supply of that good. From Sahi and Yao (1989), provided $\boldsymbol{\Lambda}$ is irreducible, there exists a unique consistent price system $\mathbf{p}(\boldsymbol{\Lambda})\in\mathbb{R}^l_{++}$. Consider the associated strategic behaviors.

The payoff function V^i of trader $i\in\mathcal{I}_\ell^F$ satisfies **A1** and may be written as

$$V^i(\mathbf{b}_\ell^i,\mathbf{b}_\ell^{-i},\mathbf{b}_{h\neq\ell},\mathbf{b}^L)\equiv U^i\left(x_1^i(\mathbf{p}(\boldsymbol{\Lambda}),\mathbf{b}_\ell^i),\ldots,\omega_\ell^i-\sum_k b_{\ell k}^i,\ldots,x_l^i(\mathbf{p}(\boldsymbol{\Lambda}),\mathbf{b}_\ell^i)\right), \tag{4}$$