Applied Soft Computing 11 (2011) 5519–5527

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

# A Stackelberg single-period supply chain inventory model with weighted possibilistic mean values under fuzzy environment

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ARTICLE INFO

*Article history:*
Received 13 September 2009
Received in revised form 11 April 2011
Accepted 1 May 2011
Available online 6 May 2011

*Keywords:*
Supply chain
Fuzzy demand
Weighted possibilistic mean value
Genetic Algorithm
Pattern Search Algorithm

ABSTRACT

This paper considers a single-period product inventory control in a distributed supply chain, which is composed of one manufacturer and one retailer and operates in the environment of uncertain market demand. A Stackelberg model with fuzzy demand is first developed, with using a *L–R* fuzzy number with a general membership function to depict the fuzzy market demand, and through adopting the weighted possibilistic mean value method to rank the retailer and the manufacturer's fuzzy profits, the risk preference of decision maker is also taken into consideration in the model. A special case where the market demand is assumed to be a triangular fuzzy number is further considered, and Genetic Algorithm and Pattern Search Algorithm are adopted to obtain the optimal solutions of the Stackelberg model. At last, numerical examples are used to illustrate the proposed model and sensitivity analyses are provided to get managerial implications.

## 1. Introduction

The single-period inventory model, which is also called as the newsboy inventory model, is one of the main building blocks of inventory theory and discussed in many literature [1]. In the real world, the single-period inventory model has wide applications in assisting the decision makers to determine the optimal order quantity [2]. The traditional literature dealing with newsboy inventory model usually assumed the market demand to be stochastic, in many real situations, however, it is very difficult to estimate the probability distribution of market demand due to the lack of historical data. Given the situations, they can only use linguistic terms, such as "the market demand is about $d_M$, but not less than $d_L$ and not larger than $d_R$", to describe the fuzzy market demand.

The fuzzy set theory, originally introduced by Zadeh [3], provides a feasible approach to deal with this kind of fuzzy circumstance problem. Since then, considerable attention has been attracted to fuzzy circumstance problem in the literature. For example, Petrovic et al. [4] proposed a newsboy-type problem with discrete fuzzy demand. Ishii and Konno [5] introduced fuzziness of shortage cost into the classical newsboy problem, where shortage cost is vague and given by a L-shape fuzzy number. Chen and Chuang [6] explored an extended newsboy problem with shortage-level constraints. Kao and Hsu [2] constructed a single-period inventory model with fuzzy demand, and utilized Yager's ranking method to gain the optimal order quantity. Li et al. [7] established two single-period inventory models in fuzzy environment, one of which assuming demand is stochastic while the holding and shortage costs are fuzzy, the other assuming the costs are deterministic but the demand is fuzzy. Dutta et al. [8] presented a single-period inventory problem in an imprecise and uncertain mixed environment, and introduced demand as a fuzzy random variable. Yao et al. [9] applied a stochastic single-period inventory management approach to analyze optimal cash management policies with fuzzy cash demand based on fuzzy integral method. Shao and Ji [10] considered a multi-product newsboy problem with fuzzy demand under budget constraint, and developed three types of models and hybrid intelligent algorithms. Dutta et al. [11] analyzed a single-period inventory model of profit maximization with a reordering strategy in a fuzzy environment. Dutta and Chakraborty [12] presented an approach to solve a single-period inventory with maximizing its expected profit in the fuzzy environment, in which the retailer has the opportunity for substitution. It is observed that all of these above studies on single-period inventory problem under fuzzy environment dealing with the optimal decision problems from the perspective of single party, however, the growing focus on supply chain management for the increasing intense competitive environment calls for a more efficient management of inventories across the entire supply chain through better coordination and more cooperation.

Different from the previous studies, in this paper, we apply Stackelberg game theory in a single-period supply chain inventory model with fuzzy demand, where the manufacturer is a leader and the retailer is a follower, which is widely reported in literature. For

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doi:10.1016/j.asoc.2011.05.007

instance, Huang and Li [13] discussed three co-op advertising models including two Stackelberg games and one cooperative game. Yao et al. [14] studied a revenue sharing contract for coordinating a supply chain consisted of one manufacturer and two competing retailers, where the manufacturer is regarded as Stackelberg leader and the retailers are seen as Stackelberg followers. Xie and Neyret [15] used Stackelberg game theory to develop co-op advertising and pricing models in manufacturer–retailer supply chains. Yu et al. [16] formulated a VMI supply chain problem with one manufacturer and multiple retailers as a Stackelberg game model, where the manufacturer leads and retailers follow to determine their own optimal product marketing and inventory policies. Yu et al. [17] discussed how a vendor can take advantage of the information that he obtains from his multiple retailers to maximize his profit by using a Stackelberg game, where implementing VMI partnership. Wu et al. [18] investigated the equilibrium behavior of two competing supply chains in the presence of demand uncertainty by taking joint pricing and quantity decisions under the manufacturer's Stackelberg game model into consideration. However, these studies on the Stackelberg games are mainly under deterministic demand or stochastic demand environment, rather than fuzzy demand.

In recent paper, Yu and Jin [19] considered a supplier–retailer channel to design the optimal return policy for the supplier under fuzzy retail price environment, and adopted the signed distance method to solve the optimization problem. Xu and Zhai [20] considered the coordination problem for single-period supply chain with fuzzy demand, and utilized Dubois and Prade's [21] ranking method to solve the optimization problem. Different from the previous related literature, we further take the risk preferences of decision makers into consideration, which making our model more consistent with real situation. We adopt the weighted possibilistic mean value method proposed by Carlsson and Fullér [22], Zhang et al. [23] to solve the fuzzy optimization problem.

Moreover, we use Genetic Algorithm (GA) and Pattern Search (PS) Algorithm to solve the final optimization problem. Genetic Algorithm is an optimization technique using the principles of evolution and heredity to obtain near-optimum solutions of difficult problems [24]. It is widely employed to solve optimization problems across different disciplines. For example, Khouja et al. [24] used GA to solve the economic lot size scheduling problem. Modal and Maiti [25] applied a GA to solve multi-item EOQ models. Gaafar [26] applied a GA to the deterministic time-varying lot sizing problem with batch ordering and backorders. Farahani and Elahipanah [27] utilized a GA to optimize the total cost and service level for just-in-time distribution in a supply chain. Pal et al. [28] developed an EPQ model with price discounted promotional demand in an imprecise planning horizon, and use Genetic Algorithm to solve the model. Pattern Search (PS) Algorithm is one of direct search methods which do not require any information about the gradient or higher derivatives of the objective function to find an optimal solution. Generally speaking, PS algorithm has the advantage of being easy to implement and computationally efficient [29] and is widely utilized to solve optimization problems in power system disciplines. Al-Sumait et al. [29] presented a new approach based on a constrained PS algorithm to solve well-known power system economic load dispatch problem with valve-point effect. Al-Othman and El-Naggar [30] used PS algorithm to solve a security constrained power system economic dispatch problem with non-smooth cost function. Alsumait et al. [31] proposed an improved algorithm based on PS algorithm to solve the Dynamic Economic Dispatch. Different from previous literature, here we use PS algorithm to solve Stackelberg game model optimization problem in supply chain management disciplines. In addition, we also compare the results obtained from GA and PS algorithm.

The remainder of this paper is organized as follows. Section 2 depicts a single-period supply chain problem with fuzzy market demand and adopts the weighted possibilistic mean value method proposed by Carlsson and Fullér [22], Zhang et al. [23] to solve the fuzzy optimization problem with considering the decision maker's risk preference. Section 3 develops the Stackelberg game models with weighted possibilistic mean value under fuzzy environment and considers two cases: $d_L \le Q \le d_M$ and $d_M \le Q \le d_R$. Section 4 considers a special triangular fuzzy demand case and use GA and PS algorithm to get the final optimal solutions. Section 5 presents some computational results including numerical examples and sensitivity analyses. Finally, conclusions are summarized in Section 6.

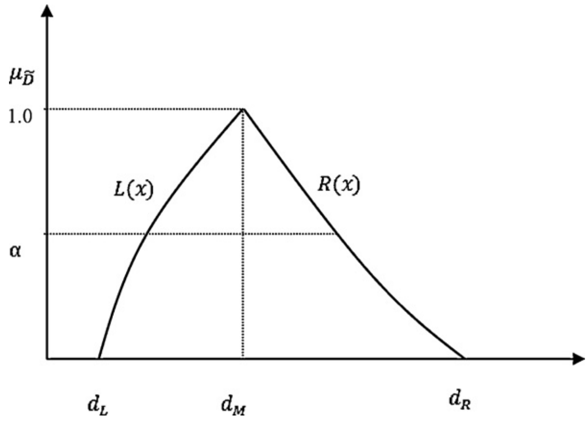

**Fig. 1.** The membership function $\tilde{D}$.

## 2. Notations and problem formulation

Consider a distributed single-period product supply chain consisted of one manufacturer and one retailer, where the manufacturer produces seasonal products and sells them to the retailer, and the retailer sells these seasonal products to consumers. For the retailer, the order quantity is $Q$, the selling price is $p$, the salvage value of unsold product is $v$, the stock out cost is zero. For the manufacturer, the wholesale price is $\omega$, the product cost is $c$. Obviously, we have $p > \omega > c > 0$. In order to avoid trivial cases, we assume $c > v > 0$.

Due to the incomplete information about the market demand, it is very difficult for the retailer to forecast the market demand precisely. However, the retailer can use fuzzy number to describe the market demand. Therefore, we assume the market demand to be a $L$–$R$ type fuzzy number $\tilde{D} = (d_L, d_M, d_R)$ with a general membership function $\mu_{\tilde{D}}$, which is defined as

$$\mu_{\tilde{D}}(x) = \begin{cases} L(x), d_L \le x \le d_M \\ R(x), d_M \le x \le d_R \\ 0, x \notin [d_L, d_R] \end{cases} \tag{2.1}$$

where $L(x)$ and $R(x)$ are the left- and right-shape functions of the fuzzy demand $\tilde{D}$, respectively. $L(x)$ is an increasing continuous function, while $R(x)$ is a decreasing function. Fig. 1 is the membership function of the $L$–$R$ fuzzy demand $\tilde{D}$.

The retailer's problem is to find the optimal order quantity $Q$ to pursue the maximum profit according to his knowledge about the market demand and a given wholesale price $\omega$. It is a classical newsboy problem, and the retailer's profit $\tilde{\Pi}_r(Q)$ can be defined as

$$\tilde{\Pi}_r(Q) = p \min\{Q, \tilde{D}\} + v \max\{0, Q - \tilde{D}\} - \omega Q \tag{2.2}$$

where $\min\{Q, \tilde{D}\}$, $\max\{0, Q - \tilde{D}\}$ respectively denote sales volume and unsold quantity.

Obviously, the retailer's profit $\tilde{\Pi}_r(Q)$ depends on the demand $\tilde{D}$ for a given order quantity $Q$ and a given $\omega$. Therefore, the retailer's profit $\tilde{\Pi}_r(Q)$ is also a $L$–$R$ fuzzy number.