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A model based on rough set theory combined with algebraic structure and its application: Bridges maintenance management evaluation

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ABSTRACT

For decision tables with a very small number of objects, this paper introduces a model combining the equivalence relations of rough set theory and algebraic structures. As an example application, we classify six-dimensional attribute vectors (maintenance quality indices) in a sample of 55 bridges from Chongqing province in China. A panel of experts has already used these data to rank the sites in terms of overall management quality, and full use is made of their decisions in training the model. Compared with the performances of a classical rough set model and a support vector machine, the new model is shown to be both feasible and more accurate.

1. Introduction

Rough set theory, introduced by Pawlak (Pawlak, 1982, 1984, 1991), is very good for data analysis. It can be used to express incomplete and/or uncertain data, and to mine all kinds of data for hidden rules. In the field of artificial intelligence, rough set theory is used to carry out attribute reduction and refine classification rules while keeping the capacity of knowledge unchanged. Nowadays it has been successfully applied in a wide range of fields (Crone, Lessmann, & Stahlbock, 2006; Inuiguchi & Miyajima, 2007; Leung, Fischer, Wu, & Mi, 2008; Pattaraintakorn & Cercone, 2008), including machine learning, decision analysis, process control, pattern recognition and data mining.

This method needs not build a very precise mathematical model in the sense of determining parameters. Instead, it is driven by the statistical properties of large data samples. However, the difficulty of attribute reduction in this theory grows exponentially with the number of attributes. If the data contain very large numbers of attributes and objects, attribution reduction becomes a NP problem. Even when the number of attributes is large and the number of objects is small, it can be very difficult to draw out characteristics and decision rules.

In recent years researchers have proposed many models based on rough set theory. These include rough membership functions (Greco, Matarazzo, & Slowinski, 2005), Bayesian rough set models (Pawlak, Wong, & Ziarko, 1998; Slezak & Ziarko, 2003), probabilistic rough set models (Pawlak, 2006; Wong & Ziarko, 1987; Yao, 2003), decision-theory rough set models (Yao & Wong, 1992), variable precision rough set models (Hong, Wang, & Wang, 2007; Pei et al., 2007; Su & Hsu, 2006; Ziarko, 1993), and fuzzy rough set models (Jiang, Wu, & Chen, 2005; Liu, Chen, Wu, & Li, 2006; Masulli & Petrosino, 2006). But there has been little research into small samples.

In this paper, we introduce a rough set decision model which combines the equivalency relations of rough set theory and algebra structure theory. When the number of objects in a decision table is very small, there is often linear dependence among its attribute vectors. This model makes use of algebraic structures (Qu, 1998) and dimension expansion (Burges, 1999; Vapnik, 2000) to assist in the reduction of rough sets to a small set of linearly independent decision vectors, cutting down the amount of calculation considerably.

As China's economy has grown, bridge construction has developed rapidly in the Chongqing province. At present, there are over 8861 bridges. While there is a drive to speed up construction, it is also necessary to improve maintenance management. As the saying goes, "construction for three years, maintenance for a hundred years." By evaluating the status of bridge maintenance and the activities of management, we can quantify the effectiveness of current procedures and develop concrete measures for raising the managerial level.

In Chongqing, technicians and managers from 55 bridges filled out a questionnaire designed for this study. A panel of experts then ranked the bridges in terms of their maintenance status based on these data. The questionnaire asked questions concerning the bridge itself and all aspects of maintenance: the number of faults,

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their positions, operating expenses, mean daily vehicle flow, bridge age, and so on. At present, maintenance evaluations of all bridges in this study are carried out by the Chengtou Bridge Management Company, following a common procedure developed at Chongqing University. The results of the questionnaire and expert evaluation were recorded in a database.

We wish to overcome the subjectivity of human judgment, yet make full use of the experts' knowledge by using artificial intelligence methods to draw effective rules from the database. Rough set theory has proven its ability to accommodate such inexactness of classification. However the number of bridges in the sample was rather small for most artificial intelligence methods. Thus, we first need to solve the problem of rough set classification in a comparatively narrow sample space.

The remaining sections of this paper are organized as follows. Section 2 introduces the new rough set model. Section 3 describes the general problem of evaluating bridge maintenance and management quality, and presents the index system for evaluations. Section 4 describes our experimental design for data collection and performance comparison. Section 5 concludes the paper and suggests directions for future work.

2. A new rough set model combined with algebra structure theory

2.1. Linear independence and algebraic mapping

Definition 1 (Hu, 1993; Ruan, 1998). A *maximal linearly independent vector set* is one which will no longer be linearly independent if any new vector is added to the set. A given vector space contains infinitely many maximal linearly independent vector sets.

It is important to note that the maximal linearly independent vector sets of a vector space are equivalent to each other; that is, any vector in the vector space can be expressed in terms of any of the maximal linearly independent vector sets.

Theorem 1 (Qu, 1998). *Consider two similar algebraic systems $V_1 = \langle A, *, \Delta, k\rangle, V_2 = \langle B, \circ, \bar{\Delta}, \bar{k}\rangle$, where '$*$' and '$\circ$' are binary operators, 'Δ' and '$\bar{\Delta}$' are monadic operators, and 'k' and '$\bar{k}$' are zero operators. Let R be a binary relationship on $V_1 \times V_2$ such that*

$$\langle a, b\rangle R \langle c, d\rangle \iff a = c \quad \forall \langle a, b\rangle, \langle c, d\rangle \in A \times B. \tag{1}$$

Then R is a coresidual relationship, and V_1 will be an isomorphic mapping of $V_1 \times V_2/R$; that is, $V_1 \times V_2/R \cong V_1$.

2.2. The new model

The main idea behind our new model is combining rough sets with maximal linearly independent vector sets to reduce the attributes of a dataset. To mine decision rules from the reduced data, it applies algebraic structure theory to a dimension expansion of the attribute table.

Consider a decision table where A is the condition attributes set and D is the decision attributes set. The table can be defined by the following mapping:

$$a_i = (a_{i1}, a_{i2}, \ldots, a_{ir}) \overset{f}{\leftrightarrow} [(a_{i1}, a_{i2}, \ldots, a_{ir}, d_{i1}, d_{i2}, \ldots, d_{il})] = [(a_i, d_i)] \tag{2}$$
$$\forall a_i \in A, \quad (a_i, d_i) \in A \times D, \quad [(a_i, d_i)] \in A \times D/R.$$

If we define $s_i = (a_{i1}, a_{i2}, \ldots, a_{ir}, d_{i1}, d_{i2}, \ldots, d_{il}) = (a_i, d_i) \in A \times D$, then the above mapping (2) can be written as

$$a_i \overset{f}{\leftrightarrow} [s_i], \quad \forall a_i \in A, \ [s_i] \in A \times D/R. \tag{3}$$

Thus, $[s_i]$ may be considered a dimension expansion on a_i.

Conclusion (1). According to Theorem 1, mapping (3) is isomorphic. That is, a random linear combination of a_i is unchanged in $[s_i]$.

2.3. Combining the rough sets model with an algebraic structure

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1r} \\ a_{21} & a_{22} & \ldots & a_{2r} \\ & & \ldots & \\ a_{t1} & a_{t2} & \ldots & a_{tr} \end{pmatrix}. \tag{4}$$

Consider a decision table condition set A, where $a_{ij} (i = 1, 2, \ldots, t; j = 1, 2, \ldots, r)$ are the values of condition attributes j giving rise to decision attribute d_i for object i. Then the decision table itself could be expressed as

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1r} & d_1 \\ a_{21} & a_{22} & \ldots & a_{2r} & d_2 \\ & & \ldots & & \\ a_{t1} & a_{t2} & \ldots & a_{tr} & d_t \end{pmatrix} \begin{matrix} \to & U_1 \\ \to & U_2 \\ \ldots & \\ \to & U_t \end{matrix} \tag{5}$$

We can build a binary relationship $R = U_i R U_j \iff a_i = a_j$, where the equivalence is between rows of matrix (4).

According to Theorem 1, R is a kind of coresidual relationship.

Objects in the decision table can be classified using R, so that $[U_i]$ represents one of the classifications as noted below.

$$[U_i] = [(a_{i1,} a_{i2}, \ldots, a_{ir}, d_i)] \quad (i = 1, 2, \ldots, l). \tag{6}$$

The number of objects in the decision table is given by l.

Our first goal is to find a maximal linearly independent vector set of the vector space a_i, which will be denoted $\sigma_i, (i = 1, 2, \ldots, m)$. Considering the characteristics of a maximal linearly independent vector set, m cannot be larger than the number of condition attributes.

When a vector of condition attributes $\tilde{\alpha}_i$ is expressed as a linear combination of the basis $\sigma_i, (i = 1, 2, \ldots, m)$, the decision attribute value corresponding to a_i also transforms. This relationship is expressed in formula (7) below. The function $f(\beta)$, where β is any vector, stands for the value of the decision attribute corresponding to a_i

$$f(\beta) = \begin{cases} k_1 d_1 + k_2 d_2 + \cdots + k_m d_m, & \text{when } \beta = k_1\sigma_1 + k_2\sigma_2 + \cdots + k_m\sigma_m, \\ d_s, & \text{when } \|\beta - \sigma_s\| = \min\limits_{1 \leqslant i \leqslant m}(\|\beta - \sigma_i\|), \end{cases} \tag{7}$$

$k_1, k_2, \ldots, k_m$ are linear coefficients.

2.4. Building the model

For clarity, we introduce the model by detailing each step.

Step 1. There are n objects in a decision table (that is, the universe $U = \{u_1, \ldots, u_n\}$). There are m condition attributes and l decision attributes, denoted $\{p_1, p_2, \ldots, p_m\}$ and $\{q_1, q_2, \ldots, q_l\}$ respectively.

Step 2. We create the binary relationship $R = U_i R U_j \iff t_i = t_j$, where t_i is a vector of condition attributes: $t_i = (p_{i1}, p_{i2}, \ldots, p_{im}), i = 1, 2, \ldots, n$. Given a vector of decision attributes $d_i = (q_{i1}, q_{i2}, \ldots, q_{il})$, we have $u_i = (p_{i1}, p_{i2}, \ldots, p_{im}, q_{i1}, q_{i2}, \ldots, q_{il}) = (t_i, d_i) \in T \times D$.

Step 3. Find a maximal linearly independent vector set, denoted $t_1, t_2, \ldots, t_r$, spanning the condition attribute vector set. $t_i = (p_{i1}, p_{i2}, \ldots, p_{ir}) (i = 1, 2, \ldots, n)$ Note that r is no larger than either n or m; that is, $r \leqslant \min\{n, m\}$.

Step 4. Build an isomorphic mapping $t_i \overset{f}{\leftrightarrow} [u_i], \forall t_i \in T, [u_i] \in T \times D/R$. Here u_i describes the value of the ith object, and $[u_i]$ stands for the classification corresponding to t_i.