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Performance Evaluation of Investment Funds with DEA and Higher Moments Characteristics: Financial Engineering Perspective

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Abstract

With the development of funds market, the research of funds performance evaluation are becoming an important topic in the field of financial engineering. In the previous research, performance evaluation of investment funds was based on some typical hypothesis, and higher moment of the assets return was mostly neglected. However, a great amount of research, both theoretical and empirical, has supported the existence of nonnormality of portfolio return and the important role of higher moments of return in the investors' utility. This has led to widespread suspicion of the validity of the traditional evaluation methodology. In this paper, data envelopment analysis (DEA) is used to evaluate the performance of the funds in the consideration of higher moments. The results show that the evaluation score is related to the utility preference of the investors, which indicates that the evaluation results are more realistic and consistent with the investors' preference.

Keywords: investment fund; performance evaluation; higher moments; data envelopment analysis; financial engineering

1. Introduction

In the 1960s, Makowitz (1952) laid the foundation of modern portfolio theory with his mean-variance(M-V) model [1]. Later, Sharpe(1964)，Lintner(1965) and Mossin(1966) proposed a capital asset pricing model (CAPM) [2-4]. Based on their research, many scholars have put forward a number of portfolio performance evaluation methods, such as Treynor index, Sharpe ratio and Jensen index. These performance evaluation methods were popular with investors and widely used in practice. However, these evaluation methods have theoretical flaw. According to Hanoch and Levy (1969) [5], Leland (1999) [6], the validity of mean-variance model must meets the following two conditions: First, the asset return is normal probability distributions; Second, the utility functions of investor preferences are quadratic. Many empirical studies show that portfolio returns are not normal distribution, which has become the fact generally is accepted by researchers.

On the other hand, some other researchs [7-9] confirmed that investors prefer the third-order central moment of returns (skewness) and disgust the fourth-order central moment (kurtosis), which means that utility functions of investors are not quadratic. The existence of higher-order moments characteristics affect the selection of portfolio. Since the mean-variance theory has the theoretical flaw, the existing performance evaluation methods which is based on mean-variance theory inherently possess the aforementioned theoretical flaw.

To measure the performance of portfolios or mutual funds, data envelopment analysis (DEA) has been used frequently to this aim. The DEA methodology has its unique advantages which don't need the hypothesis of validity of the capital market and avoid the impact that the selection of the market portfolio and risk-free rate on the evaluation results.

The purpose of this paper is to use the DEA methodology to measure the fund performance in the higher moment framework, considering the higher order moment characteristics of funds return which reflect the preference of investors. The evaluation model can take into account not only the investment cost, but also the higher moments (skewness, kurtosis), which is more consistent with the distribution of returns and utility preferences of investors. So, the evaluation results are more effective, and also overcome the problems of the former evaluation methods.

The rest of this paper is established as follows: Section 2 review the portfolio performance literature; Section 3 introudces the higher moments characteristics; Section 4 gives the evaluation model in higher moments framework; we present computational

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results using open-end funds data in Section 5 and Section 6 concludes the paper.

2. Literature review

The study of investment funds began in the 1950s. In the early performance of investment funds are evaluated mainly through two indicators, which are funds net assets and ratio of return. These indicators are easy to calculate and intuitive, but failed to systematically and quantitatively analyse the portfolio risk. In the 1960s, Markowitz (1952) proposed to take the variance of returns as risk measurement, and established the mean-variance model[1]. Based on the Markowitz's research, Sharp (1964) and Lintner(1965) and Mossion (1966) have set up the capital asset pricing model (CAPM), combining the risk with return in the linear form [2-4]. The risk-adjusted index based on CAPM, which means the return per unit risk, became the main method to evaluate the funds performance, such as [10-12]. These three indexes are ued most widely of risk-return indicators.

Some other scholars used multi-factor models based on the Arbitrage Pricing Theory (APT) to evaluate the mutual funds performance, of which three-factor model [13], four-factor model [14] are the most representative. Wang revised the TM model, HM model and GII model based on Fama-French three-factor model, and made an empirical study of timing ability of 33 China funds [15]. Tu and Zhu identified the fund's actual style by Sharpe style model, then applied Fama-French three-factor model to the funds performance evaluation [16].

The traditional methods of funds performance evaluation, although are widely used, but there are many limitations on application. First, the returns of portfolio are negative, the traditional indexes can not be used due to conflict with their original meaning; Second, as said before, CAPM-based risk-adjusted indexes have theoretical flaw in itself. Meanwhile, the hypothesis of CAPM model are too strict to meet. So these indexes are not perfect. Although multi-factor models relax the constraints, but it is hard to determine the impact factors. Moreover, the traditional methods do not consider the operation costs of the fund, which is a very important factor in performance evaluation.

To solve these problems, DEA began to be applied in fund performance evaluation. Different from the traditional methods, DEA is a non-parametric evaluation method. It does not need the hypothesis of the effectiveness of capital markets and could avoid the impact of the benchmark portfolio on evaluation result. So this approach led to the widespread concern in recent years. Murthi, Choi and Desai (1997) first used DEA to take into account the investment costs in defining a mutual fund performance index DPEI [17]. McMullen and Strong (1998) used DEA model to analyze the impact of different time horizon on fund performance [18]. Afterward, Basso and Funari (2001) extended the DPEI, and proposed a new mutual fund performance indexes that take into account a variety of transaction costs and risk measure value in DEA model [19].

Chen and Li (2001) first applied DEA in China funds performance evaluation [20]. Afterwards, a number of models based on DEA are applied to analyze china funds performance. Ding (2003) applied multiple inputs and multiple outputs DEA model to evaluate performance of investment funds[21]. Deng and Yuan (2007) established the dynamic DEA model [22]. Xu and Zhang (2009) used the input oriented BCC DEA model [23].

At present, the traditional performance evaluation methods, such as the Sharpe ratio, Jensen index and Treynor index, are used most frequently in China. These performance evaluation indicators only involve first-order moment (expectation) and second-order moment (variance) of return, based on the hypothesis that portfolio returns are normally distributed. But a large number of empirical studies have shown that asset returns are not normally distributed, but subject to asymmetric thick-tailed distribution. Meanwhile some scholars confirmed that utility function is not quadratic, and investors prefer skewness, disgust kurtosis[8]. So, modelling the portfolio selection in the higher moments framework is becoming a trend [24-26]. The main purpose of this article is to use DEA methodology to evaluate the funds performance in higher moments framework, taking into account not only the high-order moments, especially skewness and kurtosis, but also the investment costs.

3. Characteristics of higher moments

Skewness and kurtosis are almost ignored in the performance evaluation literatures. But, as said before, there exist the skewness preference and kurtosis aversion for investors,which are consistent with investors utility function.

Let r_j be a random variable representing the rate of return of asset j and λ_j be the weight of the asset j. Then, the rate of return $r(\lambda)$ of portfolio $\lambda = (\lambda_1,...,\lambda_n)$ is given by

$$r(\lambda) = \sum_j^n \lambda_j r_j \tag{1}$$

Let $\mu(\lambda)$, $v(\lambda)$, $\kappa(\lambda)$ and $\gamma(\lambda)$ be the mean, variance, skewness and kurtosis of $r(\lambda)$ where

$$\begin{aligned} \mu(\lambda) &= E[r(\lambda)] \\ v(\lambda) &= E[(r(\lambda) - \mu(\lambda))^2] \\ \kappa(\lambda) &= E[(r(\lambda) - \mu(\lambda))^3] \\ \gamma(\lambda) &= E[(r(\lambda) - \mu(\lambda))^4] \end{aligned} \tag{2}$$

Let $u(\cdot)$ be an investor's utility function. Let us consider the Taylor's series expansion of $u(r(\lambda))$ around $\mu(\lambda)$