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Financial Engineering Estimation of Minimum Risk Hedge Ratio

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Abstract

In this paper, the financial engineering minimum risk-based portfolio hedging model is first analyzed. It is then followed by the investigation on various major estimation methods for the minimum risk hedge ratio. The results revealed in the current study show that the HR obtained by the ordinary least squares (OLS) model is maximal and the out-of-sample hedging performance is the best; however, the hedging effectiveness is not sufficiently stable for both the out-of-sample and in-sample estimation.

Keywords: Financial engineering; Stock index futures; Hedge.

1. Introduction

The main purpose of the exchange of stock index futures is to avoid and resolve the systemic risks that the position of assets undergoes. That is, the hedge is carried out for the risk exposure of assets by using index futures contracts (which is also named "hedging" or "sea piano" (Wu et al., 1998)), and the future cash position is then substituted by the futures position temporarily (or the futures position is built to offset the potential risks due to the holding of cash position).

The effective use of hedging has always been the focus of academics and practitioners alike. The crucial role of the issue is the estimation of the hedge ratio (HR). Herein, the HR is defined as the relationship between the total value of futures contracts and that of stocks when the investors establish the transaction position in order to achieve the desired effect of hedging (Du, 2002), namely, HR = the total value of futures contracts / the total value of stocks. Therefore, the good or bad effect of hedging depends directly on whether the optimal hedge ratio (OHR) can accurately be calculated under various assumptions and objective functions.

A great deal of exploration involving the theoretical framework of the hedging of stock index futures is mainly devoted to the discussion of the OHR. Based on the modern portfolio hedging theory, Johnson (1960) proposed the OLS model with the minimum risk criteria. Using the Markowitz portfolio theory, Johnson (1960), Stein (1961) and Ederington(1979) regarded the futures position and the cash position as portfolios, and determined the optimal ratio between them in the condition of the minimum risk or maximum utility. According to the development of hedging,

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Ederington (1979) classified it into three categories, i.e., the simple hedging, the selective hedging and the portfolio hedging. Additionally, Kahl and Tomek (1983) advanced the Mean-Variance approach to account for the balance of benefits and risks. Howard and D'Antonio (1984) presented the optimal sharpe hedge ratio under the condition of the maximization of the utility function. Junkus and Lee (1985) empirically analyzed four kinds of the hedging strategies in light of the maximizing profits, eliminating risk, minimizing risk as well as maximizing utility. Bollerslev(1986) developed the GARCH model on the basis of the dynamics wave characteristics of financial time series. Ghosh (1993a) compared the OLS model and the EC model by empirically exploring the hedging strategies among the S&P500 Index, Dow Jones Industrial Average and NYSE Composite Index. Further, Ghosh (1993b) empirically investigated the hedging strategies of the French CAC40 stock index futures, UK FTSE100 stock index futures, Japan's Nikkei stock index futures, along with Germany's DAX stock index futures, and then concluded that the results obtained by the EC model was better than that derived by the OLS model. Chou et al. (1996) estimated and compared the hedge ratios between the conventional model and the EC model by employing the Japan's Nikkei Stock Average index futures with different time intervals. Lee et al (2009) examined four static and one dynamic hedging model by using the data from Taiwan, United States, Japan, Hong Kong , Singapore and Korean to find the optimal hedge ratios. Wang and Hsu(2010) empirically studied the hedge ratio stability of the Japan, Hong Kong and Korean index futures contracts during the Asian financial crisis and post-crisis. Krishan(2011) used daily data for the S&P CNX Nifty futures to estimate the effective hedge ratio and its hedging effectiveness of three models.

In the current paper, the author provides a deep analysis of the estimation of the minimum risk hedge ratio. For this purpose, the minimum risk-based portfolio hedging model is summarized.

2. Minimum Risk Hedge model

Based on the portfolio theory of hedging, Ederington (1979) further implemented the investigation on this topic, and indicated that the hedging objective is to minimize the variation over time in the portfolio value, and then the hedge ratio with minimum variance can be considered as the minimum risk hedge ratio, whose formula derivation is as follows:

At first, the rate of return R_s for hedgers during the period from beginning of t_0 to end of t_1 on the stock market can be given by

$$R_s = \frac{S_{t_1} - S_{t_0} + D}{S_{t_0}} \tag{1}$$

in which S_{t_0} and S_{t_1} are the stock values at the beginning of period t_0 and the end of period t_1, respectively. D is the dividends and bonuses form t_0 to t_1.

Eq. (1) implies that the dividends and bonuses will be reinvested until at the end of period t_1, during which the rate of return is regarded as the risk-free rate of return.

Then, the rate of return R_f for hedgers on the stock index futures market during the same period is formulated by Eq.(2) presented below:

$$R_f = \frac{F_{t_1} - F_{t_0}}{F_{t_0}} \tag{2}$$

in which F_{t_0} and F_{t_1} are stock index futures contracts at the beginning of period t_0 and the end of period t_1, respectively.

If R_h is defined as a rate of return of portfolio consisting of long position of stock and short position of index futures, then, according to Eqs. (1) and (2), one obtains

$$R_h = \frac{(S_{t_1} - S_{t_0} + D) - N \times (F_{t_1} - F_{t_0})}{S_{t_0}} = R_s - N \times \frac{F_{t_0}}{S_{t_0}} \times \frac{F_{t_1} - F_{t_0}}{F_{t_0}} = R_s - h \times R_f \tag{3}$$

where N is the number of futures contracts-buying, and h is the hedge ratio (HR).

Note that the variance of the rate of return of portfolio R_h is written as