

Tail Conditional Variance of Portfolio and Applications in Financial Engineering

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Abstract

The optimal portfolio selection is an important issue in financial engineering. It is well-known that downside risk measures such as TCE and CVaR only characterize the tail expectation, and pay no attention to the tail variance beyond the VaR. This is an important deficiency of measuring the extreme financial risk in engineering management, especially for insurance industry and portfolio management. In this paper, we study the optimization portfolio model based on tail conditional variance (TCV) motivated by TCE. We obtain the TCV risk of a portfolio and the explicit solution of optimal portfolio under the assumption of multivariate student t distribution. Finally, we also give an example of empirical study on China Stock Market.

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1. Introduction

The optimal portfolio selection is a classical problem in the financial engineering. It is well recognize that the measure of risk have a crucial role in portfolio optimization under uncertainty. The primitive measure of risk, the variance of returns of a portfolio, is employed in the classic mean-variance model by Markowitz. After that many new risk measures were proposed and applied in portfolio optimization. In recent years, the quantile-based downside risk measures have received much attention from practitioners and researchers. Besides the typical value-at-risk (VaR) (see Yamai and Yoshida [1], Alexander and Baptista [2]), such measures include the tail conditional expectation (TCE) and the expected shortfall (ES) defined in Benati [3]. Although VaR has undoubtedly become a standard risk measure and has been written into industry regulation, VaR's popularity does not mean that it is a sound measure. VaR ignores the magnitude of extreme or rare losses. In addition, it has been shown in Alexander et al. [4] that the problem of minimizing VaR of a portfolio can have multiple local minimizes. As response to these deficiencies of VaR, the notation of coherent risk measure was introduced in Artzner et al. [5], which is an important breakthrough for a comprehensive theory in financial risk measurement. In fact, VaR is also severely criticized that it not a coherent measure of risk due to its lack of subadditivity. That is, VaR associated with a combination of two portfolios may larger than the sum of the VaRs of the individual portfolio (see Acerbi and Tasche [6], Tasche [7], Kalkbrener [8]).

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To remedy the problems inherent in VaR, Artzner et al. [9] further proposed the use of expected shortfall, or ES for short (see also Acerbi and Tasche [8], Kamdem [10]), which is defined as the conditional expectation of loss beyond the VaR level. Other alternative measures to ES include conditional value-at-risk (CVaR) in Rockafellar and Uryasev [11, 12], tail conditional expectation (TCE) in Artzner et al. [5] and worst conditional expectation (WCE) in Benati [13]. It is noteworthy that CVaR as a coherent risk measure coincides with ES and TCE under the assumption of continuity distribution. CVaR now is rather appealing in portfolio optimization due to its attractive properties such as convexity and continuity with respect to portfolio weights, see for example [14-19]. Nevertheless, CVaR is also criticized by Lim et al. [20] for being fragile in portfolio optimization due to estimation errors.

These risk measures mentioned above undoubtedly make remarkable improvement on VaR. However they are only the linear probability weighted combination of losses beyond VaR. Some researchers argue that the higher orders of moments of the loss distribution should be considered in order to describe it comprehensively and decrease the extreme loss. For example, Cheng and Wang [21] recently proposed a new coherent risk measure based on p-norms with application in realistic portfolio optimization (see also Cheng and Wang [22,23]). Landman [24] proposed using tail conditional variance (TCV), which characterizes the variance of the loss distribution beyond some critical value. They also establish correspondingly a portfolio selection model.

The aim of this paper is to establish a portfolio selection model by considering tail conditional expectation and tail conditional variance beyond VaR simultaneously. We also will find the analytic closed solution of the associated optimal portfolio model under the multivariate student-t distribution of returns. As an application-oriented research and along a new derivation way, our approach to deriving the optimal portfolio can be regarded as the improvement on Landman [25], in which the optimal solution is based on the assumption of full rank of the constraint matrix and needs the manual partition of the constraint matrix.

This paper is organized as follows. In the next section, we provide the tail conditional variance (TCV) of a portfolio under the multivariate student-t distribution. In section 3, we formulate the optimal portfolio model under TMV defined by TCV and TCE, and derive the closed solution of the model. Section 4 illustrates some examples of Chinese market.

2. The tail conditional variance of portfolio

Consider a portfolio selection problem with n assets. The random return of the j -th asset is denoted by r_j , and the returns vector is denoted by $r = (r_1, r_2, \dots, r_n)$. Let $\mu = E(r)$ and $\Sigma = \text{Var}(r)$ be expected returns vector and covariance matrix respectively. Let ω_i be the fraction of wealth invested in asset i . We call the investment weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ a portfolio. The return of portfolio is defined by $r_\omega = r' \omega$. The expected return and the variance risk of portfolio are, respectively, given by $\mu_\omega = \mu' \omega$ and $\text{Var}(r_\omega)$

First, we recall some so-called downside risks based on quantile such as VaR, TCE, ES. Let $X = -r_\omega$ be the loss of the portfolio. Given a confidence level $1 - \alpha$, $\alpha \in (0, \frac{1}{2})$, the VaR of the portfolio ω is defined as

$$\text{VaR}_\alpha(X) = \inf\{x \mid P(X \leq x) \geq 1 - \alpha\}.$$

The tail conditional expectation (TCE) and tail conditional variance are defined respectively as the following

$$\begin{aligned} \text{TCE}_\alpha(X) &= E(X \mid X > \text{VaR}_\alpha(X)), \\ \text{TCV}_\alpha(X) &= E\left((X - \text{TCE}_\alpha(X))^2 \mid X > \text{VaR}_\alpha(X)\right). \end{aligned} \tag{1}$$

Assume that the vector of asset returns r has the multivariate elliptically distribution with the density function

$$f(x) = |\Sigma|^{-1/2} g((x - \mu)' \Sigma^{-1} (x - \mu)),$$

where g is an non-negative real function. We write $r \sim E_n(\mu, \Sigma, g)$. In particular, if the function g is defined by

$$g(u) = C_{v,n} (1 + u/v)^{-\frac{n+v}{2}}, \quad C_{v,n} = \frac{\Gamma(\frac{v+n}{2})}{\Gamma(v/2) \sqrt{(v\pi)^n}},$$

then the return vector r follows the multivariate student-t distribution with the density

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