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Optimal risk control and dividend distribution policies for a diffusion model with terminal value $\!\!\!^{\star}$

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ABSTRACT

In this paper we investigate the optimal risk control and dividend distribution problem for a diffusion model with a terminal value. Usually the insurer cedes risk by means of a reinsurance contract, and pays dividends out dynamically from the surplus. Consider that the insurer is trying to balance risk control and dividend payout in terms of reinsurance and dividend distribution policies. Then the objective is set to make a dynamic choice of reinsurance policy and dividend distribution policy, which maximizes the sum of the expected discounted dividends up to ruin time and the expected discounted terminal value.

There are two novelties in this paper. Firstly, we formulate the optimal control problem in terms of general reinsurance control policies. Each of the proportional reinsurance, the excess-of-loss reinsurance and combination of the two can be treated as a special case. It is shown that, under an expected premium principle, the dynamic excess-of-loss reinsurance is of optimal type within the general reinsurance contracts. Secondly, considering the excess-of-loss reinsurance policy and terminal value, we obtain the explicit expressions for the value function and optimal control policies by solving the HJB equation method. At the end of this paper numerical calculations are done to illustrate the influence of the terminal value on the value function and optimal policies as well.

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1. Introduction

Optimal risk control and/or the dividend distribution problem for diffusion models has been extensively studied in mathematical insurance since fifteen years ago. Usually this problem can be studied in two directions: one way is to consider the risk control problem and the dividend problem separately, while the other way is to study the risk control and dividend together in one risk model. For example, the optimal dividend control problem has been studied by Jeanblanc-Piqué and Shiryaev [1], Asmussen and Taksar [2], Paulsen and Gjessing [3], Gerber and Shiu [4,5] and Loeffen [6]. The optimal risk control problems in terms of one of proportional reinsurance, excess-of-loss reinsurance and combinational reinsurance have been studied by Højgaard and Taksar [7,8], Schmidli [9], Irgens and Paulsen [10], Zhang etc [11] and Zhou and Cai [12]. The combinational optimization of the risk control and dividend distribution has been studied by Taksar and Zhou [13], Højgaard and Taksar [14], Asmussen etc [15], Choulli etc [16], Schmidli [17] and related references therein.

The optimal control problem considering a terminal value goes back to Karatzas etc. [18]. The optimal risk control and dividend distribution problem with terminal value at time of ruin is firstly brought out in Taksar [19], since then many control problems associated with terminal value have been worked out. For example, only considering the dividend control policy,

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Boguslavskaya [20], Thonhauser and Albrecher [21] and Loeffen [22] studied the optimal dividend problem for the diffusion model, Cramér–Lundberg model and spectrally negative Lévy model respectively. As for the combinational optimization of the risk control and dividend distribution, Taksar [23] studied the optimal proportional reinsurance policy and dividend policy for a diffusion model with terminal value; Taksar and Hunderup [24] studied the optimal proportional reinsurance policy for a diffusion model with terminal value, where the dividend is uncontrollable and it is accumulated at a fixed rate of the current value of the surplus. However, the risk control and dividend distribution with terminal value has not been thoroughly solved out yet. Other cases of this problem, for example, considering risk control in terms of excess-of-loss reinsurance, are also very interesting.

In this paper we investigate the optimal risk control and dividend distribution for a diffusion model with a terminal value. A general reinsurance control policy is considered. Each of proportional reinsurance, excess-of-loss reinsurance and a combination of the two can be treated as a special case. The rest of this paper is organized as follows. In Section 2, by using the diffusion approximation to the Cramér–Lundberg model with reinsurance, we formulate the optimal control problem for a controlled diffusion model with a general reinsurance policy and a dividend policy. In Section 3, we prove that, under an expected premium principle, a dynamic excess-of-loss reinsurance is an optimal form of reinsurance within a class of general reinsurance strategies. Furthermore, in Section 4, the explicit expressions of the optimal dynamic excess-of-loss reinsurance policy and optimal dividend policy are given. We also obtain the explicit expression of the value function. At last, in Section 5, these explicit results are illustrated by numerical examples and some economic explanations are given as well.

2. A controlled diffusion risk process with reinsurance and dividend distribution policies

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t, t \ge 0\}, \mathbb{P})$ be a complete filtered probability space, on which all stochastic quantities in this paper are well defined. Here $\{\mathcal{F}_t, t \ge 0\}$ is a filtration, which satisfies the usual conditions.

In mathematical insurance the surplus of an insurance portfolio is usually described in terms of a compound Poisson process $\{X_t\}$ satisfying

$$X_t = x + ct - \sum_{i=1}^{N_t} Y_i$$
(2.1)

where c > 0 is the premium rate collected by the insurer; $\{N_t\}$ is a time-homogenous Poisson process with rate $\lambda > 0$ and N_t denotes the total number of claims occurred by time t; Y_i , i = 1, 2, ..., independent of $\{N_t\}$, denote all the claim sizes and are independent and identically distributed positive random variables with common distribution function $F(y) = 1 - \overline{F}(y) = P\{Y_i \le y\}$, finite mean $\mu^{(1)} = E[Y_i] > 0$ and finite second moment $\mu^{(2)} = E[Y_i^2] > 0$.

Define $M = \sup\{y : F(y) < 1\}$. Without loss of generalization, throughout this paper, we assume that the claim distribution has a bound support, i.e., $M < \infty$. Otherwise, given any claim distribution *F*, we can define a truncated distribution with finite support

$$F_n(x) = \begin{cases} F(x), & 0 \le x < n \\ 1, & x \ge n. \end{cases}$$

Then $F(x) = \lim_{n \to \infty} F_n(x)$.

Assume that a general reinsurance contract *R* is taken by the insurer to cede the risk of each claim, then, for claim *Y_i*, the retained risk for the insurer is $R(Y_i)$. Define correspondingly $\mu_R^{(1)} = E[R(Y_i)]$ and $\mu_R^{(2)} = E[R^2(Y_i)]$. If the reinsurance premium rate is calculated via expected value principle with loading $\theta > 0$, then the surplus of the insurance portfolio will be given by

$$X_t^R = x + (c - c^R)t - \sum_{i=1}^{N_t} R(Y_i)$$
(2.2)

where the reinsurance premium rate is

$$c^{R} = (1+\theta)\mathbb{E}\left[\sum_{i=1}^{N_{1}} (Y_{i} - R(Y_{i}))\right] = (1+\theta)\lambda(\mu^{(1)} - \mu_{R}^{(1)}).$$

In terms of the diffusion approximation of process in risk theory, for example Iglehart (1969), Grandell (1990) and Schmidli (1993), the compound Poisson process $\{X_t^R\}$ can be approximated by the diffusion process $\{Z_t^R\}$ satisfying

$$Z_t^R = x + [\theta \lambda \mu_R^{(1)} + c - (1+\theta)\lambda \mu^{(1)}]t + \sqrt{\lambda \mu_R^{(2)}}B_t$$
(2.3)

where $\{B_t\}$ is a standard Brownian motion and the two processes $\{X_t^R\}$ and $\{Z_t^R\}$ have the same drift coefficient and volatility.

Instead of (2.2), we consider the optimal control problem for the insurer basing on the diffusion model. Assume the surplus of an insurance portfolio in the presence of reinsurance is governed by (2.3). The insurer can dynamically modify

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