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Optimal risk sharing and borrowing constraints in a continuous-time model with limited commitment ☆

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Abstract

We study a continuous-time version of the optimal risk-sharing problem with one-sided commitment. In the optimal contract, the agent's consumption is a time-invariant, strictly increasing function of a single state variable: the maximal level of the agent's income realized to date. We characterize this function in terms of the agent's outside option value function and the discounted amount of time in which the agent's income process is expected to reach a new to-date maximum. Under constant relative risk aversion we solve the model in closed-form: optimal consumption of the agent equals a constant fraction of his maximal income realized to date. In the complete-markets implementation of the optimal contract, the Alvarez–Jermann solvency constraints take the form of a simple borrowing constraint familiar from the Bewley–Aiyagari incomplete-markets models.

JEL classification: C61; D86

Keywords: Risk sharing; Limited commitment; Borrowing constraints

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1. Introduction

Individuals, firms, and sovereigns alike face constraints on the amounts they can borrow. There is a large literature exploring the relation between borrowing constraints and limited contract enforcement.[1] When contract enforcement is limited, lenders face the risk of borrower default. The role of borrowing constraints is to mitigate this risk efficiently. In this paper, we contribute to this literature by studying an optimal contracting problem with limited enforcement in a tractable continuous-time framework that allows us to obtain a sharp characterization of the optimal contract as well as of the borrowing constraints that implement it.

Our analysis has two parts. In the first part, we study an optimal long-term contracting problem between a risk-neutral, fully-committed, deep-pocketed principal and a risk-averse, non-committed agent whose stochastic income process is a geometric Brownian motion. Autarky represents the agent's outside option. All information is public. In this setting, we show that under the optimal contract the agent's consumption can be represented as a strictly increasing function of the maximal level of the agent's income realized to date. In the optimal contract, therefore, the consumption path of the agent is weakly increasing and constant whenever current income is strictly below its to-date maximum but strictly increasing when income achieves a new all-time maximum. At all times, the optimal amount of risk-sharing is less than full. If the agent's preferences exhibit constant relative risk-aversion, his optimal consumption is simply given by a constant fraction of the maximal level of his income realized to date.

To see the intuition behind our characterization of the optimal contract, suppose that the principal is to deliver to the agent the level of utility exactly equal to the agent's value of autarky as of time zero. If the agent could commit to never defaulting, the optimal contract would give the agent a constant consumption flow forever. This is because the principal is risk-neutral, does not face a flow resource constraint, and discounts future payoffs at the same rate as the agent. Under this full-insurance contract, the agent's value of continuing with the contract does not change over time, i.e., remains equal to his initial autarky value. Note now that even when the agent cannot commit, the full-insurance contract does not cause the agent to default (revert to autarky) for as long as his income fluctuates below its time-zero level, i.e., for as long as the date-zero level remains the to-date maximum level attained by the agent's income process. This is because during any such time interval the agent's autarky value—being strictly increasing in income—fluctuates below the agent's initial autarky value, which means that the value of defaulting remains below the value of continuing with the contract (the agent's participation constraint is satisfied). Under the full-insurance contract, however, the agent will default as soon as his income exceeds its time-zero level—i.e., when income attains a new to-date maximum—precisely because the agent's outside option value will at that point exceed the value of continuing with the full-insurance contract. In order to prevent default, the principal has to deviate from the full-insurance contract by increasing the agent's consumption at that moment (as the agent's participation constraint binds), but not before then. So, even when the agent cannot commit, the principal will give the agent a constant consumption level for as long as the agent's income is not at its to-date maximum. The same logic applies after an all-time maximum has been realized and the agent's consumption has been increased: consumption remains constant until income hits its next all-time maximum level. And so on. Optimal consumption, therefore, is always an increasing function of the current to-date maximum level of income.

[1] Examples of contributions to this literature include Alvarez and Jermann [3], Albuquerque and Hopenhayn [2], Kehoe and Perri [13].