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Optimal risk-sharing with effort and project choice☆

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Abstract

We consider first-best risk-sharing problems in which "the agent" can control both the drift (effort choice) and the volatility of the underlying process (project selection). In a model of delegated portfolio management, it is optimal to compensate the manager with an option-type payoff, where the functional form of the option is obtained as a solution to an ordinary differential equation. In the general case, the optimal contract is a fixed point of a functional that connects the agent's and the principal's maximization problems. We apply martingale/duality methods familiar from optimal consumption-investment problems.

JEL classification: C61; J33

Keywords: First-best risk-sharing; Effort and project choice; Merton's problem; Martingale methods

1. Introduction

The problem of risk-sharing, or how to split an aggregate uncertain endowment between two or more individuals, has a long tradition in Economics, starting with Arrow [2], followed by Borch

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[4], Wilson [41] and Amershi and Stoeckenius [1], among others. All those papers consider the problem in a static setting and link it to Pareto-optimization. An influential strand of dynamic models starts with Lucas and Stokey [28], followed by Epstein [17]. They introduce recursive utilities. Risk-sharing with several agents and recursive utilities is studied in Duffie et al. [15] and Dumas et al. [16]. Dynamic optimization techniques in continuous-time are introduced by Merton [29] for a consumption/portfolio optimization problem. Technically, the choice of consumption is equivalent to the choice of effort in risk-sharing problems, while the choice of an optimal portfolio is equivalent to the choice of project. For this reason, these optimization techniques are suitable for risk-sharing problems. An early important paper in continuous-time considering a principal-agent problem with moral hazard is Holmström and Milgrom [20]. They consider exponential utility and show that the second-best contract is linear. That result has been extended by Schättler and Sung [37] and Sung [39] to the case in which the agent also chooses the project. [1] More recently, Müller [30,31] finds the full information (the "first-best") risk-sharing solution of the Holmström and Milgrom [20] problem, which is linear as well, and shows how it can be approximated by control revisions taking place at discrete times; see also Hellwig and Schmidt [19]. Bolton and Harris [3] consider a very general setting. Sannikov [36] considers the case of hidden information for more general types of utilities than Holmström and Milgrom [20]. In the same spirit, we mention Detemple et al. [14] and Cvitanić et al. [11]. Williams [40] introduces the backward stochastic differential equations approach to the problems with moral hazard, and also includes a state variable, possibly unobserved by the principal. An important recent paper is Ou-Yang [32], which considers a similar risk-sharing problem, but without independent control of the drift. His focus is on the project selection problem, applied to the choice of a portfolio by a money manager acting for an investor. Finally, we mention Larsen [27], which solves numerically the case with power utilities for the linear, portfolio delegation case, for contracts which depend only on the final value of the portfolio.

Most of the continuous-time papers use the techniques first introduced in finance by Merton [29], or the more recent martingale methods, which were suggested as an alternative way to solve the optimal consumption/portfolio optimization problem. Martingale techniques were developed initially by Pliska [34], Cox and Huang [5] and Karatzas et al. [22], and presented in great generality in Karatzas and Shreve [24]. See also Cvitanić and Zapatero [12] for a more elementary exposition.

In this paper we consider a very general first-best risk-sharing framework, with effort and project selection. The continuous-time papers with full information mentioned above are particular cases of our model. As in those papers, we take advantage of the similarity of the problem to the setting considered in Merton [29]: optimal consumption in this paper would be equivalent to optimal effort, and optimal portfolio would correspond to project selection. In particular, we use the martingale methodology to solve the problem. This approach allows us to consider general utility functions for both agents and characterize the optimal contract. In principle, it can be applied to general semimartingale (and not just Markovian diffusion) settings. Martingale methods work particularly well in a complete markets setting. For our problem, that means the possibility to fully control the volatility of the aggregate wealth process ("project selection"). In addition, we

[1] These papers are related to a different strand of the literature that considers risk-sharing with "hidden action" in dynamic discrete-time settings: Spear and Srivastava [38] characterize the general solution, using dynamic programming principles; Phelan and Towsend [33] present a numerical algorithm to solve a general set of dynamic problems with hidden action; DeMarzo and Fishman [13] apply the dynamic programming results of the former to a large number of problems affecting the dynamics of a firm.