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Optimal risk taking in an uneven tournament game with risk averse players

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ABSTRACT

We analyze the optimal choice of risk in a two-stage tournament game between two players that have different concave utility functions. At the first stage, both players simultaneously choose risk. At the second stage, both observe overall risk and simultaneously decide on effort or investment. The results show that those two effects which mainly determine risk taking – an effort effect and a likelihood effect – are strictly interrelated. This finding sharply contrasts with existing results on risk taking in tournament games with symmetric equilibrium efforts where such linkage can never arise. Conditions are derived under which this linkage leads to a reversed likelihood effect so that the favorite (underdog) can increase his winning probability by increasing (decreasing) risk which is impossible in a completely symmetric setting.

1. Introduction

In rank-order tournaments, players compete for given prizes. The best performing player (e.g. the one with the highest output) receives the highest prize, the second best performer gets the second highest prize and so on. Distribution of prizes according to relative performance creates considerable incentives for all contestants since ex ante each player wants to be declared winner of the tournament. There are many examples for rank-order tournaments in practice: sales representatives compete for bonuses which have been fixed in advance (Murphy et al., 2004), workers take part in job-promotion tournaments (Baker et al., 1994), athletes participate in sports contests (Szymanski, 2003), lawyers compete in litigation contests (Wärneryd, 2000), firms and individuals invest in external or internal rent-seeking contests (Gibbons, 2005), managers receive relative performance pay (Gibbons and Murphy, 1990), firms spend resources for advertising in winner-take-all markets (Schmalensee, 1976), there are research tournaments (Schöttner, 2008) and even tournaments in broiler production (Knoeber and Thurman, 1994).

Theoretic models which analyze players' behavior in rank-order tournaments[1] typically focus on the effort or investment decision of contestants: The more input a player chooses relative to his opponents the higher will be his probability of winning the tournament. However, in real tournaments players can often also decide on the risk of their behavior. For example, before firms choose their advertising expenditures, they can decide on whether introducing a new product (high risk) or not (low

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[1] See, among many others, Lazear and Rosen (1981) and O'Keeffe et al. (1984). For a survey on tournaments and contests see Konrad (2007).

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risk). In many tournaments, contestants first have the choice between using a standard technique (low risk) or switching to a new technique (high risk); thereafter they decide on effort or, more generally, on input to win the tournament.

This paper addresses such a two-stage tournament game with risk taking at the first stage and effort or input choice at the second stage. We focus on heterogeneous and risk averse workers so that interior pure-strategy equilibria at the effort stage are always asymmetric. This asymmetric outcome has important consequences on those effects which mainly determine risk taking in tournaments: First, the choice of risk influences the equilibrium efforts at the second stage (*effort effect*). Second, risk also influences the players' probabilities of winning (*likelihood effect*). Previous work on risk taking in two-stage tournaments has only considered symmetric equilibria at the effort stage 2. There, the effort effect and the likelihood effect were completely separate. However, this outcome will no longer hold, if players' utility functions at least slightly differ. Then both effort effect and likelihood effect are strictly interrelated which can be crucial for risk taking. In particular, we can show that in this situation the *favorite* (*underdog*), i.e. the player with the higher (lower) probability of winning in equilibrium, may prefer high risk (low risk) in order to maximize his winning probability which is impossible under symmetry. This new effect is labeled *reversed likelihood effect* since it works just contrary to the original likelihood effect. We also consider the role of risk aversion. Whereas under mean-variance preferences the more risk averse player always prefers maximum risk this result need not hold for alternative utility functions.

So far there is only a small number of papers which also address the problem of risk taking in two-stage tournaments. Hvide (2002) focuses on the case of homogeneous players. Since equilibrium efforts are always identical in his setting and the winning probability of each player is always one half irrespective of the risk level, basically there is no likelihood effect in the Hvide-model. Since equilibrium efforts are monotonically decreasing in risk, each player chooses maximum risk according to the effort effect so that players exert minimum effort at the second stage of the game. In the setting of Hvide (2002), maximizing risk at the first stage works like an implicit collusion for the effort choices at the second stage.

When introducing heterogeneity between the players, there are two possibilities in principle. Following O'Keeffe et al. (1984) we can differentiate between *unfair* and *uneven* tournaments. In an unfair two-person tournament, players choose identical efforts so that again we have a symmetric equilibrium like in the case of homogeneous contestants. However, one of the players has a lead and, hence, a higher probability of winning. In an uneven tournament, only asymmetric equilibria exist since players have different cost-of-effort functions or different preferences of winning (i.e. the subjective tournament prizes or the respective utilities of the players differ). Kräkel and Sliwka (2004) combine the problem of risk taking with unfair tournaments. In their setting, equilibria at the effort stage are always symmetric like in Hvide (2002). However, now both effort and likelihood effect are important. Whether the players prefer high or low risk in order to reduce effort costs depends on the magnitude of the favorite's lead. If the lead is small (large) both players are interested in choosing a high (low) risk in order to destroy overall incentives at the second stage according to the effort effect. Concerning the likelihood effect, there is an unambiguous result due to the symmetric equilibrium: The favorite (underdog) maximizes his winning probability by choosing low (high) risk. In this paper, we address risk taking by considering uneven tournaments in which only asymmetric interior equilibria can exist at the second stage. In this setting, it can be shown that the effort effect and the likelihood effect are strictly interrelated, which sharply contrasts with the findings for symmetric equilibria in Hvide (2002) and Kräkel and Sliwka (2004). This linkage between the two effects can lead to a so-called reversed likelihood effect under which the favorite – and not the underdog – prefers high risk in order to maximize his winning probability.

There are some other papers that also deal with risk taking in tournaments. In the models by Gaba and Kalra (1999), Hvide and Kristiansen (2003) and Taylor (2003), players can solely decide on risk taking in the tournament; hence there is no effort effect and no possible linkage with the likelihood effect. Other papers analyze risk taking empirically. For example, Becker and Huselid (1992) consider individual behavior in stock-car racing. Their results show that drivers take more risk if tournament prizes and prize spreads are large. Knoeber and Thurman (1994) find out that more able contestants tend to choose less risky strategies. However, their empirical analysis is not based on a theoretical model that allows effort choices to react on risk. The findings of Brown et al. (1996) and Chevalier and Ellison (1997) point out that presumable losers – contrary to presumable winners – prefer high risks in tournaments between mutual fund managers. Finally, the paper by Grund and Gürtler (2005) on professional soccer confirms the previous findings that leading players or teams (players that lie behind) prefer low-risk (high-risk) behavior. However, neither of the empirical papers addresses the effort effect.

The paper is organized as follows. The next section introduces the model. Section 3 considers the effort stage. Section 4 focuses on risk taking at the first stage of the game; it contains the main results. In particular, the reversed likelihood effect is highlighted in Sections 4.2 and 4.3. Section 5 concludes.

2. The basic model

Two risk averse players A and B participate in a two-stage tournament. Player i's ($i = A, B$) production or performance function is given by

$$q_i = e_i + \varepsilon_i \tag{1}$$

where e_i denotes investment or effort chosen by player i. ε_A and ε_B are exogenous noise terms. The density of the composed random variable $\varepsilon_j - \varepsilon_i$ ($i, j = A, B; i \neq j$) is denoted by $g(\cdot)$ and the corresponding cumulative distribution function by $G(\cdot)$ which is assumed to be continuous and twice differentiable. The density $g(\cdot)$ is assumed to be symmetric around its unique