Available online at www.sciencedirect.com
ScienceDirect
Journal of Economic Theory 133 (2007) 152–176
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Optimal risk sharing with background risk

Rose-Anne Dana[a], Marco Scarsini[b,*,1]

[a] *CEREMADE, Université Paris Dauphine, Place du Maréchal de Lattre de Tassigny, F-75775 Paris Cedex 16, France*
[b] *Dipartimento di Statistica e Matematica Applicata, Università di Torino, Piazza Arbarello 8, I-10122 Torino, Italy*

Received 14 June 2004; final version received 13 October 2005
Available online 28 November 2005

Abstract

This paper examines qualitative properties of efficient insurance contracts in the presence of background risk. In order to get results for all strictly risk-averse expected utility maximizers, the concept of "stochastic increasingness" is used. Different assumptions on the stochastic dependence between the insurable and uninsurable risk lead to different qualitative properties of the efficient contracts. The new results obtained under hypotheses of dependent risks are compared to classical results in the absence of background risk or to the case of independent risks. The theory is further generalized to nonexpected utility maximizers.

JEL classification: D52; G22

Keywords: Insurance; Efficient contracts; Incomplete markets; Stochastically increasing

1. Introduction

Since the early work of Borch [8], many authors have considered the problem of the optimal sharing of risk between an insurer and an insured. In particular, Arrow [1–3] showed that if the premium set by a risk-neutral insurer depends only on the actuarial value of the policy offered and is fair, then the optimal policy for a risk-averse von Neumann–Morgenstern insured is full insurance. If the premium includes a factor loading, then the optimal policy contains full insurance above a deductible. This result plays an important role in the literature since it shows that the insured's decision can be brought down to a one-dimensional problem, the choice of the optimal deductible (or equivalently of the optimal premium). In particular, it allows for comparative static results.

* Corresponding author.
E-mail addresses: dana@ceremade.dauphine.fr (R.-A. Dana), marco.scarsini@unito.it (M. Scarsini)
URL: http://web.econ.unito.it/scarsini

[1] The work of this author was partially supported by MIUR-COFIN.

0022-0531/$ - see front matter © 2005 Elsevier Inc. All rights reserved.
doi:10.1016/j.jet.2005.10.002

Raviv [39] reconsidered the problem under a more general set of assumptions: the insurer can be risk averse and the premium is a convex function of the supplied insurance. He showed that if the marginal cost is greater than 1, then the efficient policy of a risk-averse von Neumann–Morgenstern insured entails coinsurance above a deductible. Furthermore, if the insurer is risk averse and if the cost of insurance equals the supplied insurance, then the efficient policy is coinsurance. It may also be checked that if the cost is too high the insured takes no insurance.

These results have at least two important implications: first, insurer and insured wealths are comonotone, each being nonincreasing function of the risk (in other words, there is risk sharing). Second, if there are no costs, then there are no deductibles, and, if there are costs, then there are deductibles. Hence costs explain deductibles.

A drawback of this analysis is that it assumes that markets are complete. As already mentioned in [12,13,22,42], first insurers prefer to cover different sources of risk by different contracts and next, some risks such as war, floods, earthquakes, market risks, and human capital are not insurable. Hence, it seems necessary to reconsider insurance problems under the assumption of background risk. Furthermore, the problem of insurance in the presence of background risk arises in the pricing of climatic options (the risk to be insured is a climatic risk and the background risk is the financial risk, see for example [6,7]). It also arises when an insured faces a sequence of risks over time and chooses at each date an insurance contract that depends on the forthcoming risk and on her history.

It is well-known that the presence of background risk in wealth has an effect on the demand for other risks. Several papers have considered different risk postures of decision makers in the presence of background risk, among them [14,19,21,24,26,29,30,37,38,40]. An extended treatment of background risk and relevant references can be found in [23].

Insurance with background risk has been considered in various settings, since the early work of Doherty and Schlesinger [13], who first addressed the problem of insurance demand with background risk in a two-state economy. The problem has been reconsidered by a number of authors (see the survey paper [41] which discussed the case of proportional coinsurance under independent and dependent background risk). This trend in the literature has focused for the most part on comparing demand when there is background risk to demand when there is no background risk. Eeckhoudt and Kimball [15] have considered background risk of loss such that a higher level of insurable risk implies a less risky distribution of the background risk loss in the sense of third degree stochastic dominance. In the setting of an exchange economy where agents have HARA utilities, Franke et al. [20] studied the effect of introducing an independent background risk.

Gollier [22] first examined the problem of efficient contracts when there is background risk under the assumptions that the insurer is risk neutral, and that the premium is a function of the expected indemnity, with marginal increase greater than 1. Assuming two dependent sources of risks such that the uninsurable risk has zero conditional expectation given any value of the insurable risk, he showed that if a higher level of insurable risk implies a more risky distribution of the background risk, then the deductible rule does not apply anymore. He showed also that if the insured is prudent, then the optimal insurance contract entails a disappearing deductible. Efficient contracts when there is background risk were also considered in [34,46] in slightly different models, in which the risks are independent, but the loss in revenue is not additive.

This paper shows that the shape of the optimal insurance contract crucially depends on the type of dependence among the insurable and noninsurable risks.

The optimal contract results, to be stated in the sequel, concern a large class of decision makers, and entail no parametric assumptions of the distribution of risks. All strictly risk-averse expected utility maximizers will be considered, namely, all agents whose von Neumann–Morgenstern