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Exact and heuristic methods for a class of selective newsvendor problems with normally distributed demands ☆

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ABSTRACT

In this paper we study a class of selective newsvendor problems, where a decision maker has a set of raw materials each of which can be customized shortly before satisfying demand. The goal is then to select which subset of customizations maximizes expected profit. We show that certain multi-period and multi-product selective newsvendor problems fall within our problem class. Under the assumption that the demands are independent and normally, but not necessarily identically, distributed we show that some problem instances from our class can be solved efficiently using an attractive sorting property that was also established in the literature for some related problems. For our general model we use the KKT conditions to develop an exact algorithm that is efficient in the number of raw materials. In addition, we develop a class of heuristic algorithms. In a numerical study, we compare the performance of the algorithms, and the heuristics are shown to have excellent performance and running times as compared to available commercial solvers.

1. Introduction

In this paper we study a class of selective newsvendor problems (SNPs) that generalizes the classical newsvendor problem by incorporating a degree of flexibility regarding the shape of the demand distribution faced by an inventory manager into the decision making progress. In particular, our generic model considers a set of raw materials that can be customized immediately prior to satisfying demand. The raw materials could be physically different items, but also simply a single item in different periods. The processes by which customization can take place are identical for each of the raw materials; e.g., we can think of coloring, packaging, etc., or preparation for satisfying demand in a particular market or segment. The selection flexibility lies in the ability to invest in a collection of customization methods or options. This model is applicable in several important practical settings. For example, it could be used as a prescriptive model for a manufacturer who has the opportunity to make an optimal selection, but also as a tool for gaining managerial insights for a manufacturer who is considering a modification of their selection. In addition, the type of selection models that we will consider in this paper also often appear as a subproblem for solving assignment, facility location, or other more comprehensive models (see, e.g., [5,8,11,12,16]).

As mentioned above, our problem is based on the single-period newsvendor problem (see [10] for a general overview of stochastic inventory models). Eppen [4] considered a generalization of this problem to multiple locations, which allows for a reduction in the expected costs associated with variability in demand by considering all locations together and planning for aggregate demand. This observation has more recently led to research on market selection problems where the manufacturer has a choice of a set of markets (e.g., locations) that may be served. Taaffe et al. [15] first introduced a selective newsvendor problem, there defined as a market selection problem with independent and normally distributed demands for each market. The authors demonstrate that such a market selection problem can be solved efficiently using a sorting algorithm that ranks the markets according to the ratio of net expected revenue to demand variance. Taaffe et al. [17] studied the case of all-or-nothing demand distributions, and Taaffe and Chahar [14] and Chahar and Taaffe [2] included risk as an additional objective. In related work, deterministic market selection problems with Economic Order Quantity (EOQ) [7] and Economic Lot Sizing (ELS) costs [18] were studied, while Geunes et al. [6] reviewed demand selection and assignment problems. Chen and Zhang [3] and Huang and Sošić [9] study allocations of profits for a newsvendor game. Testing if an allocation of profits is in the core of the game is closely related to selection problems.

In the context of our paper, the basic selective newsvendor problem introduced and studied by Taaffe et al. [15] can be viewed as a customization selection problem corresponding to only a single

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raw material. In this paper we (i) discuss a wider range of applications of this general problem class and (ii) generalize that model to account for multiple raw materials. Taaffe et al. [15] showed that the case of a single raw material can be solved efficiently using a sorting-based algorithm. We develop an exact solution approach for the more general and computationally much more challenging class of selective newsvendor problems with multiple raw materials. Finally, we propose a class of heuristics inspired by the exact solution approaches and show, through extensive computational tests, that particular implementations of these are both effective and efficient.

The remainder of the paper is organized as follows. Section 2 describes the model and some applications. In Section 3 we develop exact and heuristic solution approaches, while Section 4 provides a variety of computational results. Finally, in Section 5 we summarize our results and describe future research directions.

2. Problem formulation

2.1. Notation and general model

Let $A=\{1,\ldots,a\}$ be a set of *raw materials* that may be *customized* shortly before satisfying demand in one of different ways, indexed by the set $N=\{1,\ldots,n\}$. The problem that we will study in this paper is to determine a subset of customizations and a set of order quantities for the raw materials that maximize expected profit. To this end, define the binary decision variables $z_i=1$ when customization i is selected and $z_i=0$ otherwise ($i\in N$), as well as the continuous decision variables Q_j ($j\in A$), denoting the raw material order quantities. For convenience, let $Q=(Q_j; j\in A)^\top$ and $z=(z_i; i\in N)^\top$.

Let the random variable $\mathbf{D}_{ij}$ denote the demand for raw material $j\in A$ customized according to $i\in N$, and let $\mathbf{D}_j=(\mathbf{D}_{ij}: i\in N)^\top$ ($j\in A$) denote the corresponding demand vectors. Furthermore, let

- F_i be the fixed charge associated with selecting customization method $i\in N$;
- $\overline{r}_{ij}$ be the unit revenue for satisfying demand for raw material $j\in A$ customized according to $i\in N$;
- c_j be the unit cost of raw material $j\in A$, and $c=(c_j; j\in A)^\top$;
- v_j be the unit salvage value for raw material $j\in A$;
- e_j be the unit expediting cost for raw material $j\in A$.

For convenience, we let $\mu_{ij}=E[\mathbf{D}_{ij}]$ (for $j\in A$, $i\in N$) and define $\overline{r}_i=\sum_{j\in A}\overline{r}_{ij}\mu_{ij}-F_i$ (for $i\in N$). To ensure that the problem is meaningful, we assume that $e_j>c_j>v_j$ for all $j\in A$. The expected profit as a function of the decision variables can then be expressed as

$$\overline{r}^\top z-c^\top Q+\sum_{j\in A}v_jE[(Q_j-\mathbf{D}_j^\top z)^+]-\sum_{j\in A}e_jE[(\mathbf{D}_j^\top z-Q_j)^+]$$

It is easy to see that, given values for the selection variables, this problem decomposes into a traditional newsvendor problem for each raw material. This means that the optimal order quantity for raw material $j\in A$ is the $\rho_j\equiv((e_j-c_j)/(e_j-v_j))$-fractile of the distribution of demand for that raw material.

In the remainder of this paper, we will follow Taaffe et al. [15] and assume that $\mathbf{D}_{ij}\sim \text{Normal}(\mu_{ij},\sigma_{ij}^2)$ (for $(i,j)\in N\times A$). In addition, we assume that, for each $j\in A$, the elements of $\mathbf{D}_j$ are independent; however, the vectors $\mathbf{D}_j$ may be dependent. Clearly, both the normality and independence assumptions may be violated in practice. In fact, this assumption is relaxed in Strinka and Romeijn [13] where approximation algorithms for such problems are developed. However, in this paper we will limit ourselves to the special case since under these assumptions the optimization problem takes on an interesting form. We then develop exact and heuristic approaches for a more general class of problems that may be of independent interest.

Taaffe et al. [15] show that normality and independence of the demand vector (for fixed $j\in A$) can be used to further simplify the expected profit function. In particular, letting $s_{ij}=\sigma_{ij}^2$ ($i\in N, j\in A$), $s_j=(s_{ij}: i\in N)^\top$, $r_i=\sum_{j\in A}(\overline{r}_{ij}-c_j)\mu_{ij}-F_i$, and $r=(r_i: i\in N)^\top$, we obtain the optimization problem

$$\max_{z\in\{0,1\}^N} r^\top z-\sum_{j\in A}f_j(s_j^\top z) \qquad \text{(P)}$$

where, for all $j\in A$, $f_j(x)=K_j\sqrt{x}$ with $K_j=(c_j-v_j)\Phi^{-1}(\rho_j)+(e_j-v_j)L(\Phi^{-1}(\rho_j))$ a nonnegative constant, where Φ denotes the c.d.f. of the standard normal distribution and L denotes the associated loss function. This model generalizes the basic selective newsvendor problem (SNP) as introduced by Taaffe et al. [15]. As noted earlier in this paper, their problem is a special case of (P) where $a=1$ and n is interpreted as a collection of markets that may or may not be entered by the supplier. Despite the fact that (P) is still a convex maximization problem for general values of a and therefore there exists an optimal extreme point (i.e., binary) solution to its continuous relaxation, this generalization makes the mathematical programming problem (P) considerably more challenging to solve, since a sorting approach can no longer be applied in general. Although in our application the functions f_j have the form given above, all of our results in fact apply more generally to the case where these functions are concave and nondecreasing. Moreover, without loss of generality we will assume that $r_i>0$ for all $i\in N$ (since it is easy to see that an optimal solution exists for which $z_i=0$ for all $i\in N$ for which $r_i\le 0$).

2.2. Examples

In this section we will discuss several multi-period and multi-item selective newsvendor problems that can be formulated as special cases of (P).

Multi-item selective newsvendor problems. Of course, the generic description of our problem class can be viewed as a multi-item selective newsvendor problem where, by definition, we have to select a common set of customizations for the raw materials. However, note that if we can select a different set of customizations for each raw material then the problem decomposes by raw material. In other words, for each raw material we obtain an optimization problem that selects the set of customizations for that raw material. (We will refer to this case as C1.)

Multi-period selective newsvendor problems. A generalization of the basic SNP described above to a multi-period setting considers the demands for a single product in a set of M different markets over a horizon of time periods indexed by the set T, denoted by the random variables D_{it} ($i\in M$; $t\in T$). We assume that all selection and ordering decisions have to be made at the start of the horizon. In this setting, we must decide (i) whether or not the set of selected markets can change between periods and (ii) whether or not inventory or backlogging is allowed between periods. In the remainder of this section, we will show how this problem reduces to one of the form (P) under a number of different simplifying assumptions.

With respect to the market selection, we will consider models that do not allow the set of selected markets to change between periods as well as models that do allow for costless changes in the set of selected markets. With respect to the inventory carryover and backlogging, we assume that these are either costless or not allowed.

T1. If inventory carryover and backlogging as well as changes in the market selection are allowed and costless, then demand