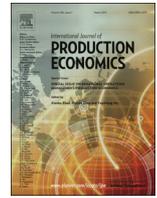




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## Newsvendor problem with random shortage cost under a risk criterion

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## ABSTRACT

We study profit maximization vs risk approaches for the standard newsvendor problem with uncertainty in demand as well as a generalized version with uncertainty in the shortage cost (as often applies in practice). We consider two well-known risk approaches: Value-at-Risk (VaR) included as a constraint and Conditional Value-at-Risk (CVaR). We first derive the explicit expressions of the optimal solution with uncertainty of shortage cost under different risk measures and then perform a numerical analysis to quantify the effect of uncertainty in shortage cost and risk measures. For the standard newsvendor problem, we find that the optimal quantity under CVaR is always lower than that under the VaR constraint, which in turn is lower than the order quantity that maximizes the expected profit. Insightful explanations for this result are that: (a) a higher degree of risk aversion drives the newsvendor to order fewer products, increasing the likelihood that all will be sold; (b) this effect is stronger for the CVaR approach as this does not consider the expected profit at all. Another interesting and counter-intuitive observation for the CVaR approach is that a higher retail price may lead to a smaller order quantity, as fewer items need to be sold in order to attain a sufficient profit. These results show that one should be careful in employing the CVaR risk measure for newsvendor type problems. The results remain valid if the shortage cost becomes uncertain. However, increased uncertainty of this type does improve the relative profitability under the CVaR approach by increasing the order quantity under that criterion whilst there is no effect under the other criteria.

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## 1. Introduction

As a fundamental problem in inventory control, the newsvendor problem has been studied for a long time and applied in a broad array of business settings. The classic newsvendor model assumes that managers select an order quantity to maximize the expected profit under the risk neutral assumption. However, Schweitzer and Cachon (2000) provide experimental evidence suggesting that inventory managers are risk averse for high-value products. Thus, the assumption of risk neutrality is not always applicable. A number of authors have therefore considered risk related objectives, such as utility functions (e.g., Lau, 1980; Agrawal and Seshadri, 2000; Chen et al., 2007); the mean-variance objective function (e.g., Chen and Federgreen, 2000; Wu et al., 2009); VaR and downside risk (e.g., Özler et al., 2009; Gan et al., 2005), CVaR and coherent measures (e.g., Ahmed et al., 2007; Chen et al., 2009). An overview of the key results is provided in the next section.

VaR and CVaR as financial risk measures have emerged and been widely used in recent years. In this paper, we focus on VaR and CVaR as the risk measures of the downside risk. The standard VaR criterion maximizes the minimum profit that is attained with a certain probability, e.g., with a probability of 95% the realized profit is at least an amount to be maximized. More formally, the standard VaR maximizes the lower end of a one-sided confidence interval (Artzner et al., 1999). A clear disadvantage of the standard VaR criterion is that it purely considers risk but not the expected profits. Although many inventory manager may be risk averse, they still strive to attain a high (expected) profit. To reflect this, VaR can be used as a constraint, e.g., with a probability of 95% the realized profit is at least some fixed amount. See also Gan et al. (2005). Another way to trade-off the expected profit and risk is by using the CVaR criterion. Loosely formulated, the CVaR maximizes the profit to be achieved after subtracting the expected amount by which the actual profit may fall short of that amount, multiplied by a risk factor. A formal definition is provided in Section 4. See also Rockafellar and Uryasev (2000, 2002) for more details. And we refer interested readers to Qin et al. (2011) for summaries of the ordering policies of newsvendors with various risk preferences.

In this paper, we consider both the VaR constraint and the CVaR approach. Indeed, an important contribution of our research is to show that the choice of risk measures has an important effect on

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the inventory decision. We find that the CVaR typically (and always with a constant shortage cost) leads to much smaller order quantities and lower profits than the VaR constraint approach. This result, which to the best of our knowledge has gone unnoticed in the literature, shows that risk criteria should be carefully selected for inventory decisions.

Another main contribution is that we consider uncertainty in both demand volume and shortage cost. The classical newsvendor problem, reflecting the majority of the inventory control literature, only considers the first type of uncertainty. In practice, however, shortage costs are often variable and unknown. For example, [Corsten and Gruen \(2003\)](#) conduct a worldwide study of consumer responses to out-of-stocks in fast-moving consumer goods industry. They categorize five primary responses to out-of-stock, i.e., 31% of customers choose to buy the same item at another retailer, 15% of customers choose to buy later at the same retailer, 19% of customers choose a substitute with same brand for a different size or type, 26% of customers choose a substitute with a different brand, 9% of customers do not purchase the item. Since all these five responses result in different amounts of profit losses for a retailer, the shortage cost is uncertain. Another cause of randomness in shortage cost, especially in a service logistics environment, is that the cost of dealing with a shortage may vary considerably. For instance, if some spare part is needed but not available for completing a repair, then that part can sometimes be cannibalized from a partially manufactured product or a product that is collected/returned after use. If this is indeed possible, then the shortage cost is relatively low. If, however, the repair cannot be completed and the customer is offered a new (comparable) product, then the shortage cost is much higher. We remark that even if the shortage cost is fairly constant, it may be difficult to estimate, which also creates uncertainty.

The uncertainty of the shortage cost in inventory problems has received research attention in recent years. Some authors defined shortage costs as a fuzzy variable. For example, [Ishii and Konno \(1998\)](#) introduced fuzzy shortage cost into traditional discrete random newsvendor problem; [Wee et al. \(2009\)](#) constructed a multi-objective joint replenishment deteriorating items inventory model, where the demand and shortage cost were assumed to be fuzzy variables; [Petrović et al. \(1996\)](#) investigated the newsvendor problem with fuzzy demand and fuzzy costs. However, this paper presents the first analysis of the newsvendor problem with a random shortage cost under different risk measures. We find that uncertainty of the shortage cost does not affect the order quantity under the traditional criterion and under the VaR constraint. However, it results in an increase of the order quantity under the CVaR criterion.

The remainder of this paper is organized as follows. In the next section, we review the literature on the risk-averse newsvendor problem. Then, in [Section 3](#), we derive the optimal order decision for the classic newsvendor model with random shortage cost. In [Section 4](#), we describe the risk measure CVaR and get the optimal order quantity under the CVaR criterion. [Section 5](#) considers the newsvendor problem with a VaR constraint and derives the optimal order quantity for a special case. [Section 6](#) presents numerical results and discusses the effects of different risk measures and uncertainty in the shortage on order quantity. [Section 7](#) concludes with a general discussion and directions for future research. All proofs can be found in the online appendix.

## 2. Literature review

Three main streams of modelling risk-averse newsvendor problems can be identified in the literature: utility functions, mean variance approach, VaR/CVaR. Next, we review key contributions for

each stream separately, after which we draw some general conclusion.

The first stream is related to utility function. [Eeckhoudt et al. \(1995\)](#) considered the newsvendor model with a risk-averse decision maker where risk is measured by a utility function. They examined the effects of risk aversion on a prudent newsvendor without shortage costs. [Schweitzer and Cachon \(2000\)](#) provided a complete investigation of the relationship between the newsvendor's profit-maximizing order quantity and optimal order quantities under various alternative objectives. They showed that a loss-averse newsvendor would order strictly less than a risk-neutral newsvendor. [Wang and Webster \(2009\)](#) continued their research by including a shortage cost. They found that a loss-averse newsvendor may order more than a risk-neutral newsvendor if shortage cost is not negligible, and that the optimal order quantity may increase in the wholesale price and decrease in the retail price.

The second is related to mean-variance objective function. [Chen and Federgruen \(2000\)](#) studied the newsvendor problem using the mean-variance framework. They showed that the variance of the stochastic profit is a monotone increasing function of the order quantity. Without shortage cost, the optimal risk-averse order quantity is shown to be smaller than the risk-neutral order quantity and to decrease in the degree of risk aversion. [Wu et al. \(2009\)](#) showed that this result does not extend in general to the case with a positive shortage cost.

The third is to use the risk measure of VaR and/or CVaR. [Gan et al. \(2005\)](#) incorporated the VaR concept to a newsvendor problem with a downside risk constraint for a single product. They studied the channel coordination without shortage cost where the supplier is risk-neutral and the retailer is constrained by a downside risk. [Özler et al. \(2009\)](#) considered the single period newsvendor problem with VaR constraints in the multi-product case. They derived the exact profit distribution function for the two-product newsvendor problem and developed an approximation method for the profit distribution of multi-product case. However, they did not incorporate shortage cost into the models. [Ahmed et al. \(2007\)](#) studied the multi-period newsvendor problem without shortage cost where the objective function is a coherent risk measure. They showed that the structure of the optimal solution of the risk-averse model is similar to that of the risk-neutral case. Without shortage cost, [Jammernegg and Kischka \(2007\)](#) compared the ordering policy and its corresponding performance measures, such as the cycle service level, under the CVaR and the expected profit approach. They found that a risk-averse (risk-seeking) newsvendor orders less (more) than a risk-neutral newsvendor does. [Xu and Li \(2010\)](#) discussed the newsvendor problem under a hybrid model with a combination of the CVaR and the expected profit. The authors showed that the optimal order quantity under the CVaR criterion is increasing in the shortage cost; incorporating a substantial shortage cost, the optimal order quantity of the risk-averse newsvendor may be larger or less than that of the risk-neutral newsvendor. [Choi et al. \(2011\)](#) considered a multiproduct risk-averse newsvendor under the law-invariant coherent measures of risk. For identical products with independent and generally distributed demands, they showed that increased risk aversion leads to decreased orders. [Jammernegg and Kischka \(2012\)](#) gave a comparative analysis of CVaR, VaR, and the mean-CVaR rule in the newsvendor problem without shortage cost. They analyzed the impact of these risk measures on the distribution functions of the profit, the optimal order quantity, and the optimal expected profit. In particular, they showed that the CVaR newsvendor orders less than the VaR newsvendor. [Katariya et al. \(2013\)](#) gave a comparative analysis of expected utility, mean-variance and CVaR in the newsvendor problem with shortage cost. They showed that the optimal order

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