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Spatial competition with concave transport costs

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Abstract

We study the location-then-price game played by two firms in a circular market when consumers face non-linear transport costs. We show that for any convex transport cost function there exists a concave one such that the location-then-price games induced by these functions are strategically equivalent. Further, we provide a sufficient condition to guarantee that a similar equivalence result holds under oligopolistic competition among equidistant firms.

Keywords: Spatial competition; Equilibria; Oligopoly

JEL classification: C72; D43; L13

1. Introduction

Salop's (1979) circular market model has been widely used to study localized competition (see, for instance, Anderson, 1986 and Economides, 1989). Competition on the circular market can be interpreted as competition around a lake or as competition by companies offering daily services at a particular time of the day. As with the linear market model, contributions using the circular model have

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generally considered specific convex transport cost functions whenever a multi-stage game is proposed.

The purpose of this paper is to show that the convexity or concavity of the transport cost function is not relevant for the competitive decisions of the firms when the market is circular. More precisely, we show that for a given increasing convex transport cost function there exists an increasing concave one such that the location-then-price games induced by these functions are strategically equivalent.

In a circular market with two firms competing to sell their products, a consumer located at x, $x \in [0, 1)$, purchasing a product from firm located at x_i at price p_i obtains utility

$$u(x, x_i) = -p_i - F(z_i), \tag{1}$$

where z_i denotes the distance between x and x_i and where F stands for the transport cost of travelling z_i, with $F(0) = 0$. Clearly, if both firms always charge the same prices then a transformation in the transport cost function from convex to concave results in a monotone transformation in the consumers utilities. Thus, consumer theory tells us that the consumers decisions will not be modified by this transformation, and, consequently, the firm's competitive decisions will be not modified either. However, whenever firms charge different prices then a transformation in the transport cost does not induce a monotone transformation in the consumers' utilities. Nevertheless, the symmetry of the circular market allows one to find for any consumer x facing C, another consumer $\bar{x}$ facing T whose utility is a monotone transformation of the utility of x. Consumer $\bar{x}$ facing T will hence buy from the same firm that consumer x does, facing C. The above argument cannot be extended to the linear market model.

The main result in the paper is hence an equivalence (or duality) result showing that the game induced by a given convex transport cost C is strategically equivalent to the game induced by a certain concave transport cost T. This result may have some interest inasmuch as until now it has not been known how to incorporate concave transport costs into the analysis of spatial competition. Furthermore, the result has several implications. On one hand, it ensures that, for the circular market, neither the existence of equilibrium nor the pattern of product differentiation rely on the convexity of transport costs. On the other hand, it allows one to translate any existence or uniqueness result for particular convex cost functions into the corresponding result for the appropriate concave cost functions.

In a previous work (see De Frutos et al., 1999) we show that only two cost functions from the linear quadratic family — $C(z) = z^2$ and $T(z) = z - z^2$ — ensure existence of a perfect equilibrium in pure strategies. One might think that this result deprives our current results of generality. We do not take this view. Rather we think that the contribution of this paper is to show that relaxing the widely used assumption of convexity in the transport cost has no impact on the results. In