Int. J. Production Economics 117 (2009) 338–359

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

An overview of theory and practice on process capability indices for quality assurance

Chien-Wei Wu [a,*], W.L. Pearn [b], Samuel Kotz [c]

[a] Department of Industrial Engineering and Systems Management, Feng Chia University, 100, Wenhwa Road, Seatwen, Taichung 40724, Taiwan
[b] Department of Industrial Engineering and Management, National Chiao Tung University, Taiwan
[c] School of Science and Engineering, George Washington University, Washington, DC, USA

ARTICLE INFO

Article history:
Received 11 April 2007
Accepted 2 January 2009
Available online 10 December 2008

Keywords:
Expected relative loss
Fraction non-conforming
Process capability indices
Process consistency
Process relative departure
Quality assurance

ABSTRACT

Process capability indices (PCIs), C_p, C_a, C_{pk}, C_{pm}, and C_{pmk} have been developed in certain manufacturing industry as capability measures based on various criteria, including process consistency, process departure from a target, process yield, and process loss. It is noted in certain recent quality assurance and capability analysis works that the three indices, C_{pk}, C_{pm}, and C_{pmk} provide the same lower bounds on the process yield. In this paper, we investigate the behavior of the actual process yield, in terms of the number of non-conformities (in ppm), for processes with fixed index values of $C_{pk} = C_{pm} = C_{pmk}$, possessing different degrees of process centering. We also extend Johnson's [1992. The relationship of C_{pm} to squared error loss. Journal of Quality Technology 24, 211–215] result formulating the relationship between the expected relative squared loss and PCIs. Also a comparison analysis among PCIs is carried out based on various criteria. The result illustrates some advantages of using the index C_{pmk} over the indices C_{pk} and C_{pm} in measuring process capability (yield and loss), since C_{pmk} always provides a better protection for the customers. Additionally, several extensions and applications to real world problem are also discussed. The paper contains some material presented in the Kotz and Johnson [2002. Process capability indices—a review, 1992–2000. Journal of Quality Technology 34(1), 1–19] survey but from a different perspective. It also discusses the more recent developments during the years 2002–2006.

1. Introduction

Understanding the structure of a process and quantifying process performance no doubt are essential for successful quality improvement initiatives. Process capability analysis has become—in the course of some 20 years—an important and well-defined tool in applications of statistical process control (SPC) to a continuous improvement of quality and productivity. The relationship between the actual process performance and the specification limits (or tolerance) may be quantified using suitable process capability indices. Process capability indices (PCIs), in particular C_p, C_a, C_{pk}, C_{pm} and C_{pmk}, which provide numerical measures of whether or not a manufacturing process is capable to meet a predetermined level of production tolerance, have received substantial attention in research activities as well as an increased usage in process assessments and purchasing decisions during last two decades. By now (2006) there are several books (on different levels) cited in the references, which provide discussions of various PCIs. A number of authors have promoted the use of various process capability indices and examined (with a various degree of completeness) their properties.

* Corresponding author. Tel.: +886 4 24517250x3626; fax: +886 4 24510240.
E-mail address: cweiwu@fcu.edu.tw (C.-W. Wu).

0925-5273/$ - see front matter
doi:10.1016/j.ijpe.2008.11.008

The first process capability index appearing in the engineering literature was presumably the simple "precision" index C_p (Juran, 1974; Sullivan, 1984, 1985; Kane, 1986). This index considers the overall process variability relative to the manufacturing tolerance as a measure of process precision (or product consistency).[1] Another index C_a, a function of the process mean and the specification limits, referred to as an "accuracy" index, is geared to measure the degree of process centering relative to the manufacturing tolerance (see, e.g., Pearn et al., 1998). This index is closely related to an earlier measure originally introduced in the Japanese literature (see Section 3). Formally:

$$C_p = \frac{USL - LSL}{6\sigma}, \quad C_a = 1 - \frac{|\mu - m|}{d}, \tag{1}$$

where μ is the process mean, σ is the process standard deviation, USL and LSL are the upper and the lower specification limits, $d = (USL{-}LSL)/2$ is the half specification width related to the manufacturing tolerance and $m = (USL{+}LSL)/2$ is the midpoint between the upper and lower specification limits. Due to its simplicity, C_p cannot provide an assessment of process centering (targeting). The index C_{pk}, on the other hand, takes both the magnitude of process variance and the process departure from the midpoint m into consideration. It may be written as $C_{pk} = C_p \times C_a$ a product of the two basic indices C_p and C_a. The standard definition is

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} = \frac{d - |\mu - m|}{3\sigma}. \tag{1$'$}$$

As alluded above the index C_{pk} was developed because C_p does not adequately deal with cases where process mean μ is not centered (the mean does not equal to the midpoint m). However, C_{pk} by itself still cannot provide an adequate measure of process centering. That is, a large value of C_{pk} does not provide information about the location of the mean in the tolerance interval $USL{-}LSL$. The C_p and C_{pk} indices are appropriate measures of progress for quality improvement situations when reduction of variability is the guiding factor and process yield is the primary measure of a success. However, they are not related to the cost of failing to meet customers' requirement of the target. A well-known pioneer in the quality control, G. Taguchi, on the other hand, pays special attention on the loss in product's worth when one of product's characteristics deviates from the customers' ideal value T.

To take this factor into account, Hsiang and Taguchi (1985) introduced the index C_{pm}, which was also later proposed independently by Chan et al. (1988). The index is motivated by the idea of squared error loss and this loss-based process capability index C_{pm}, sometimes called the Taguchi index. The index is geared towards measuring the ability of a process to cluster around the target, and reflects the degrees of process targeting (centering). The index C_{pm} incorporates the variation of production items relative to the target value and the specification limits which are preset in a factory. The index C_{pm} is defined as

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d}{3\tau}, \tag{2}$$

where as above $USL{-}LSL$ is the allowable tolerance range of the process, $d = (USL{-}LSL)/2$ is the half-interval length, and $\tau = [\sigma^2 + (\mu - T)^2]^{1/2}$ is a measure of the average product deviation from the target value T. This index C_{pm} can also be expressed as a function of the two basic indices C_p and C_a, explicitly $C_{pm} = C_p/\{1 + [3C_p(1 - C_a)]^2\}^{1/2}$. The quantity $\tau^2 = E[(X - T)^2]$ combines two variation components: (i) variation relative to the process mean (σ^2) and (ii) deviation of the process mean from the target ($(\mu - T)^2$).

Pearn et al. (1992) proposed the process capability index C_{pmk}, which combines the features of the three earlier indices C_p, C_{pk} and C_{pm}. The index C_{pmk} (motivated by the structure of C_{pk} (1)′) alerts the user whenever the process variance increases and/or the process mean deviates from its target value. The index C_{pmk} has been referred to as the third-generation capability index, and is defined as

$$C_{pmk} = \min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\} = \frac{d - |\mu - m|}{3\sqrt{\sigma^2 + (\mu - T)^2}}. \tag{3}$$

Comparing the pair of indices (C_{pmk}, C_{pm}), analogously to (C_{pk}, C_p), we obtain the relation $C_{pmk} = C_{pm} \times C_a = (C_{pm} \times C_{pk})/C_p$. Consequently, C_{pmk} can be expressed as $C_{pmk} = C_pC_a/\{1 + [3C_p(1 - C_a)]^2\}^{1/2}$ in terms of the "elementary indices". More recently, Vännman (1995) has proposed a superstructure $C_p(u, v) = (d - u|\mu - m|)/\{3[\sigma^2 + v(\mu - T)^2]^{1/2}\}$ of capability indices for processes based on normal distribution, which includes C_p, C_{pk}, C_{pm} and C_{pmk} as particular cases. By setting u, $v = 0$ and 1, we obtain the four indices $C_p(0, 0) = C_p$, $C_p(1, 0) = C_{pk}$, $C_p(0, 1) = C_{pm}$, and $C_p(1, 1) = C_{pmk}$. These indices are effective tools for process capability analysis and quality assurance. Two basic process characteristics: the process location in relation to its target value, and the process spread (i.e. the overall process variation) are combined to determine formulas for these capability indices. The closer the process output is to the target value and the smaller is the process spread, the more capable the process is. The first feature (closeness to the target) is reflected in the denominator while the second one (the process spread) appears in the numerators of these four indices. In other words, the larger the value of a PCI, the more capable is the process. In this paper, all derivations are carried out assuming that the process is in a state of statistical control and the characteristics under investigation arise from a normal distribution. Moreover, the target value is taken to be the midpoint of the specification limits: $T = m$ (which is common in practical situation) unless stated otherwise.

During the last two decades many authors have promoted the use of various PCIs and examined them

[1] We have not been able to discover any publications on C_p between Juran (1974) and Sullivan (1984).