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On the modeling and solution algorithm for the reverse logistics recycling flow equilibrium problem

Huey-Kuo Chen [a,*], Huey-Wen Chou [b], Yi-Chang Chiu [c]

[a] *Department of Civil Engineering, National Central University, Taiwan*
[b] *Department of Information Management, National Central University, Taiwan*
[c] *Department of Civil Engineering and Engineering Mechanics, The University of Arizona, United States*

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Abstract

This paper presents a study that characterizes, formulates, and solves the reverse logistic recycling flow equilibrium (RLRFE) problem. The RLRFE problem is concerned with the recycling channel in which recyclable collectors, processors, landfills, and demand markets form a multi-tiered network to process the recycled material flows from sources destined either for landfills or demand markets. Motivated by a government policy making or enterprise conglomerate recycling system design and operation needs, the RLRFE problem is elaborated from a system-optimal perspective using the variational inequality (VI) approach. For each origin–destination (OD) pair, the corresponding equilibrium conditions are established as a variation of the Wardrop second principle. In light of demand and cost function interactions, a nested diagonalization solution (ND) algorithm is proposed that gradually transforms the RLRFE problem into a traffic assignment model. To address multiple landfills in the recycling network and to understand how a variable-demand problem can be analyzed as a fixed-demand problem, we propose a supernetwork representation of the RLRFE problem. A numerical analysis on a test case illustrates the model formulation and the proposed algorithm.

Keywords: Reverse logistics; Recycling; Network equilibrium; Variational inequality; Nested diagonalization method; Conversion factor; Policy making

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1. Introduction

Reverse logistics includes all the logistics processes associated with a product/service after the point of sale, with the goals to either maximize value at the end of the goods' original useful life, to satisfy regulatory requirements, or to fulfill "green logistics" while maintaining financial viability for agents in this logistics process. Types of activity common with reverse logistics includes: logistics, warehousing, repair, refurbishment, recycling, e-waste, after market call center support, reverse fulfillment, field service and several others. The

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[*] Corresponding author. Tel.: +3 4227151x34115; fax: +3 4252960.
E-mail addresses: ncutone@ncu.edu.tw (H.-K. Chen), hwchou@mgt.ncu.edu.tw (H.-W. Chou), chiu@email.arizona.edu (Y.-C. Chiu).

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Notations

a	any link in the reverse supply chain network, $a \in \{ri, i, ij, j, js, (r, \lvert S\rvert+1), (i, \lvert S\rvert+1), (j, \lvert S\rvert+1)\}$
$c_{ri,1}$	unit cost function faced by source r for transaction between source r and collector i
$c_{ri,2}$	unit cost function faced by collector i for transaction between source r and collector i
c_i	unit recycling cost associated with collector i
$c_{ij,1}$	unit cost function faced by collector i for transaction between collector i and processor j
$c_{ij,2}$	unit cost function faced by processor j for transaction between collector i and processor j
c_j	unit processing cost function associated with processor j
$c_{js,1}$	unit cost function faced by processor j for transaction between processor j and demand market s
$c_{js,2}$	unit cost function faced by demand market s for transaction between processor j and demand market s
$\hat{c}_{ri,1}$	marginal cost function faced by source r for transaction between source r and collector i
$\hat{c}_{ri,2}$	marginal cost function faced by collector i for transaction between source r and collector i
$\hat{c}_i$	marginal recycling cost associated with collector i
$\hat{c}_{ij,1}$	marginal cost function faced by collector i for transaction between collector i and processor j
$\hat{c}_{ij,2}$	marginal cost function faced by processor j for transaction between collector i and processor j
$\hat{c}_j$	marginal processing cost function associated with processor j
$\hat{c}_{js,1}$	marginal cost function faced by processor j for transaction between processor j and demand market s
$\tilde{c}_p^{Ks}$	cost associated with route p from super-origin K to demand market s
$d_s(\boldsymbol{\rho})$	the demand function in market s
g_a	amount of processed/semi-processed waste flow on link a (subproblem)
i	collector designation, $i \in I$, $I = \{1, 2, \ldots, i, \ldots, \lvert I\rvert\}$
j	processor designation, $j \in J$, $J = \{1, 2, \ldots, j, \ldots, \lvert J\rvert\}$
K	super-origin designation
p	route designation
q_a	amount of processed/semi-processed waste flow on link a
q^{Ks}	total amount of product shipped from super-origin K to demand market s
Q	amount of processed/semi-processed waste transaction between each pair of tiers, $Q = \{q_{ri}, q_i, q_{ij}, q_j, q_{js}, q_{r,\lvert S\rvert+1}, q_{i,\lvert S\rvert+1}, q_{j,\lvert S\rvert+1}\}$
$(\overline{Q} \setminus \bar{q}_a, q_a)$	processed/semi-processed waste flows Q in the network are temporarily fixed at a certain level, except for the subject product flow q_a
r	source designation, $r \in R$, $R = \{1, 2, \ldots, r, \ldots, \lvert R\rvert\}$
s	demand market designation, $s \in S$, $S = \{1, 2, \ldots, s, \ldots, \lvert S\rvert\}$
$\lvert S\rvert + 1$	landfill designation
β_a	conversion factor associated with link a
λ	step size
$\bar{\rho}_{r,\lvert S\rvert+1}$	fixed price associated with disposing one unit of the waste in the landfill for source r
$\bar{\rho}_{i,\lvert S\rvert+1}$	fixed price associated with disposing one unit of the waste in the landfill for collector i
$\bar{\rho}_{j,\lvert S\rvert+1}$	fixed price associated with disposing one unit of the waste in the landfill for processor j
ρ_s	processed waste price in demand market s
$(\bar{\boldsymbol{\rho}} \setminus \bar{\rho}_s, \rho_s)$	processed waste prices $\boldsymbol{\rho}$ in all demand markets temporarily being fixed at a certain level except for subject demand market s
Ω	feasible region of the original VI problem
Ω_d	feasible region of the diagonalized VI subproblem, associated with diagonalized demand functions

entire reverse logistic process can be as complex as the forward logistics since it can be characterized by broad definitions and configurations. Several recent studies address the system configuration and operations