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Influence of dependency between demands and returns in a reverse logistics system

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ABSTRACT

We consider two reverse logistics systems where returned products are as good as new. For the first system, the product return flow is independent of the demand flow. We prove that the optimal policy is of base-stock type and we establish monotonicity results for the optimal base-stock levels, with respect to the system parameters (arrival rate, production rate, return rate, production cost, lost-sale cost, return cost and holding cost). We also provide an efficient algorithm to compute the optimal base-stock level. For the second system, demands and returns are strongly correlated: a satisfied demand induces a product return after a stochastic return lead-time, with a certain probability. When the return lead-time is null, we extend the results obtained for the first system. When the return lead-time is positive, the optimal control problem is more complex and we do not prove that the optimal policy is of base-stock type. However, we provide a framework to analyse base-stock policies. Finally, we carry out a numerical study on many scenarios to investigate the impact of ignoring dependency between demands and returns. We observe that ignoring this dependency yields to non-negligible cost increase.

1. Introduction

Recycling and recovery of used products have drawn attention of companies for several years, not only for ecological reasons, but also for legal and economical ones. At the same time, customers return more and more items to the producers for numerous reasons (DeCroix and Zipkin, 2005). The returned items constitute return flows that must be taken into account. The management of this material flow, opposite to the conventional supply chain flow, is addressed in the rapidly expanding field of reverse logistics (Fleischmann et al., 1997). From a logistic point of view, and regardless of why they occur, product returns complicate the management of an inventory system (DeCroix et al., 2005). First, returns represent an exogenous inbound material flow causing an increase of the inventory between replenishments. Second, returned products – when recovered – give another alternative supply source for replenishing the serviceable inventory (Fleischmann and Kuik, 2003). Several researches investigated the influence of product returns on inventory control. For an overview, we refer the reader to Fleischmann et al. (1997).

Most of the models do not take into account the relation between returns and demand (see for instance Fleischmann et al., 2002). de Brito and Dekker (2001) have explored the assumptions generally made in stochastic models with product returns such as the assumption of independence between returns an demand. They conclude that it is necessary to break with this traditional assumption. Most of the models with product returns that are investigated assume a total, or partial, independence between demanded items and returned ones. This is owed to the great complexity which could be led by the relaxation of this hypothesis.

Among the authors that consider the dependency relation of returns with demand, Simpson (1978) considers a repairable inventory problem where the dependency between the demand process and return process is allowed only in the same period. Kiesmueller and van der Laan (2001) develop a periodic review model with constant return and procurement lead-times. They compare the case of dependent returns with the case of independent returns and obtain numerically that the average cost is smaller in the dependent case. Cheung and Yuan (2003) consider a continuous review model with Poisson demand, exponential return lead-time and instantaneous procurement lead-time. They adopt an (s,S) inventory policy and develop an algorithm to compute the optimal replenishment parameters. However, none of these models investigate the impact of neglecting correlation between demand and returns.

In this paper, we relax the instantaneous procurement lead-time assumption of Cheung and Yuan (2003). We use the framework of make-to-stock queues (Veatch and Wein, 1996; Ha, 1997) to model a stochastic and capacitated production process by a single exponential server. This framework allows us to thoroughly characterize the optimal control policy. We consider two make-to-stock systems. In the first one, demands and returns are independent Poisson processes. We prove that the optimal policy is of base-stock type. We establish monotonicity results for the optimal base-stock levels, with respect to

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the system parameters (arrival rate, production rate, return rate, production cost, lost-sale cost, return cost and holding cost). We then compute analytically the average cost for a given base-stock level and provide properties of the average cost with respect to the base-stock level. In the second model, demands and returns are correlated: a satisfied demand induces a product return with a certain probability after a stochastic return lead-time. We extend the results of the first model when the return lead-time is null. This special case is interesting for several reasons. It is a tractable case where the impact of ignoring dependence between returns and demands is maximum. It also provides a good approximation for short-term returns. When the return lead-time is positive, the structure of the optimal policy is more complex to establish and depends on whether or not we can observe which sold products will be returned. The assumption of observability is not realistic in most of situations and we will not consider this case. When there is no observability, the production decisions can be based only on the inventory level. For this case, we restrict the analysis to base-stock policies and we suggest a numerical procedure to compute the optimal base-stock policy.

Finally, we carry out a numerical study to investigate the impact of ignoring dependency between demand and returns. We first compare the system with independent returns to the system with dependent returns. Then we suggest a heuristic for the system with dependent returns, based on the system with independent returns. We begin by investigating thoroughly the zero return lead-time before looking at the influence of return lead-time.

The remainder of this paper is organized as follows. Section 2 (resp., Section 3) presents the formulation and results for the system with independent (resp. dependent) demand and return processes. These results allow us, in Section 4, to study the impact of ignoring correlation of demand and returns. Finally, in Section 5, we conclude and suggest future research.

2. Model with returns independent of demands

We first consider a simple model where product returns and demand are independent stochastic processes. We will refer to this case as Model 1.

2.1. Formulation

We consider a make-to-stock system producing a single item. The supplier can decide at any time to produce or not this item. The unit production cost is c_p. The processing time is exponentially distributed with mean $1/\mu$ and completed items are stored in a serviceable product inventory, where they incur an holding cost c_h per unit per unit time. Demands for those items arrive according to a Poisson process with rate λ. A demand that cannot be fulfilled immediately, when the inventory is empty, is lost and incurs a lost-sale cost c_l including image cost, penalty cost, etc. We assume that the production cost c_p is smaller than the lost-sale cost c_l, otherwise it is optimal to idle production all the time.

We also suppose that there are random returns of items (Fig. 1) that are immediately available to serve customer demand. The inventory is common to new and returned products which are considered as good as new. In this first model, returns occur according to a Poisson process, independent of the demand process, with rate δ. Let $p=\delta/\lambda$ be the proportion of returned products if all demands were satisfied, and $q=1-p$. We emphasize that p is larger than the proportion of returned products since some demands are not satisfied. We assume that the return rate is smaller than the demand rate, i.e. $\delta<\lambda$ (or equivalently $0\leq p<1$). In an infinite planning horizon, this assumption clearly holds if returns are linked to previously satisfied demands. It also guarantees the stability of the stock level. A return incurs a return cost c_r including

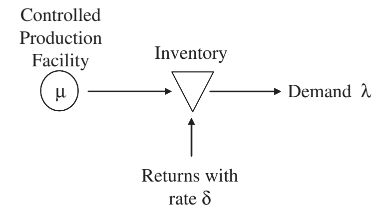

Fig. 1. Returns independent of demand.

logistics return costs (repackaging, handling) and possibly the reimbursement of the customer. The state of the system can be summarized by $X(t)$, the stock level at time t (including new and returned products).

A policy π specifies, at any time, when to produce or not. The objective of the supplier is to find the optimal policy minimizing expected discounted costs over an infinite time horizon. We denote by $\beta>0$ the discount rate.

2.2. Characterization of the optimal policy

We prove in this section that the optimal policy is a base-stock policy.

Definition 1. A base-stock policy, with base-stock level S, states to produce whenever the stock level is strictly below S and not to produce otherwise.

The problem of finding the optimal policy can be modelled as a continuous-time Markov decision process (MDP). We restrict our analysis to stationary markovian policies since there exists an optimal stationary markovian policy (Puterman, 1994).

We define $v^{\pi}(x)$ as the expected total discounted cost associated to policy π and initial state x. We seek to find the optimal policy π^* minimizing $v^{\pi}(x)$ and we let $v^*(x)=v^{\pi^*}(x)$ denote the optimal value function:

$$v^*(x)=\min_{\pi} v^{\pi}(x)$$

We denote by β the discount factor. Then, we can uniformize (Lippman, 1975) the MDP with rate $\gamma>\beta+\lambda+\mu+\delta$ and the optimal value function can be shown to satisfy the following optimality equations:

$$v^*(x)=Tv^*(x),\quad \forall x\in\ \mathbb{N}$$

where $\mathbb{N}$ is the set of natural numbers and operator T is a contraction mapping defined as

$$Tv(x)=\frac{1}{\gamma}[c_h x+\mu T_1 v(x)+\lambda T_2 v(x)+\delta T_3 v(x)+(\gamma-\beta-\lambda-\mu-\delta)v(x)] \tag{1}$$

and

$$T_1 v(x)=\min[v(x),v(x+1)+c_p]$$

$$T_2 v(x)=\begin{cases} v(x-1) & \text{if } x>0 \\ v(x)+c_l & \text{if } x=0\end{cases}$$

$$T_3 v(x)=v(x+1)+c_r$$

Operator T_1 corresponds to production decisions while operator T_2 corresponds to demand arrivals. Operator T_3 is associated to product return events.

To prove that the optimal policy is of base-stock type, it is sufficient to show that the optimal value function $v^*(x)$ is convex in the stock level x. A function v in $\mathbb{N}$ is said to be convex if and only if $\Delta v(x)=v(x+1)-v(x)$ is non-decreasing in x. We will also use the