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Optimal production–inventory strategies for a HMMS-type reverse logistics system

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Abstract

The aim of the paper is to find optimal inventory policies in a reverse logistics system with special structure. It is assumed that demand is a known continuous function in a given planning horizon and return rate of used items is a given function. There is a constant delay between the using and return process. We investigate two stores. The demand is satisfied from the first store, where the manufactured and remanufactured items are stored. The returned products are collected in the second store and then remanufactured or disposed. The costs of this system consist of the quadratic holding costs for these two stores and the quadratic manufacturing, remanufacturing and disposal costs.

The model is represented as an optimal control problem with two state variables (inventory status in the first and second store) and with three control variables (rate of manufacturing, remanufacturing and disposal). The objective is to minimize the sum of the quadratic deviation from described inventory levels in stores and from described manufacturing, remanufacturing and disposal rates. In this form, the model can be considered as a generalization of the well-known Holt et al. (Planning Production, Inventories, and Work Forces, Prentice-Hall, Englewood Cliffs, NJ, 1960) model with two warehouses. After solving the problem, we give some numerical examples to represent the optimal path in dependence of the demand rates.

Keywords: Remanufacturing; Reverse logistics; Optimal policies; Optimal control

1. Introduction

Reverse logistics is a term for manufacturing of materials and remanufacturing of from market returned reusable materials. The demand is to be satisfied with new manufactured (produced) and the remanufactured products, so there is no difference between manufactured and remanufactured items. In order the environment with waste not to burden, or the use of minerals to spare, it is a good opportunity to reuse the used items, if it is economical.

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Deterministic one-product reverse logistics systems are investigated extensively with the optimal control theory in the last year. Optimal control characterizes the optimal trajectory in time. The application of modern control theory supplies qualitative informations about the optimal path and decision rules in time.

Minner and Kleber (1999) and Dobos (1999) have investigated a linear reverse logistics problem with linear cost structure and with disposal. The results of these examinations are that the optimal manufacturing–remanufacturing strategy is extremal. It means that the optimal trajectory contains always constraints either on inventory levels or on the control variables. A similar result was shown by Kleber et al. (2000) for a multiple reuse model with linear cost structure.

One product reverse logistics models with convex manufacturing, remanufacturing and disposal costs were analyzed by Kistner and Dobos (2000) and by Dobos and Kistner (2000). The first contribution examines a reverse logistics model without disposal. This model can be interpreted as a multi-product generalization of the Arrow and Karlin (1958) model. After characterizing the optimal path, a forward algorithm was given by Kistner and Dobos to construct the optimal manufacturing–inventory control system. Dobos and Kistner (2000) have generalized the convex model with disposal activity.

The aim of the paper is to investigate a HMMS-type (Holt et al., 1960) reverse logistics model. It is assumed that the demand is a known continuous function in a given planning horizon and the return rate of used items is a given function. There is a constant delay between the using and return process. We investigate two stores. The demand is satisfied from the first store, where the manufactured and remanufactured items are stored. The returned products are collected in the second store and then remanufactured or disposed off. The costs of this system consist of the quadratic holding costs for these two stores and the quadratic manufacturing, remanufacturing and disposal costs.

The paper organizes as follows. The model is presented in Section 2. The next section supplies the necessary and sufficient conditions for optimality. Then some characteristics of the model is given. In Section 5 we present some numerical examples to illustrate optimal trajectories. And last we summarize the results.

2. The model

We examine a two-store reverse logistics model with continuous disposal. The problem can be represented as an optimal control model with two state variables (inventory status in the first and second store) and with three control variables (rate of manufacturing, remanufacturing and disposal). The objective is to minimize the sum of the holding costs in the stores and costs of the manufacturing, remanufacturing and disposal. The following parameters are used in the model:

T	length of the planning horizon,
$S(t)$	the rate of demand, continuous differentiable,
τ	the delay of the return, nonnegative,
r	proportion of return in store 2 ($0 \leqslant r \leqslant 1$),
$R(t)$	the rate of return, continuous differentiable, $R(t) = rS(t-\tau)$,
$\bar{I}_1(t)$	inventory size goal level in store 1,
$\bar{I}_2(t)$	inventory size goal level in store 2,
$\bar{P}_{\mathrm{m}}(t)$	manufacturing rate goal level,
$\bar{P}_{\mathrm{r}}(t)$	remanufacturing rate goal level,
$\bar{P}_{\mathrm{d}}(t)$	disposal goal level,
h_1	the inventory holding coefficient in the first store,
h_2	the inventory holding coefficient in the second store,
c_{m}	production cost coefficient for the remanufacturing,