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Volatility risk premium in the interest rate market: Evidence from delta-hedged gains on USD interest rate swaps

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ABSTRACT

This study examines whether the interest rate market compensates for volatility risk. It demonstrates that the delta-hedged gain (DHG) method introduced by Bakshi and Kapadia (2003) shows the existence and sign of DHG in the interest rate swap markets where they use measures different from what Bakshi and Kapadia assumed. This finding is applied to the USD interest rate swap and swaption market. The result shows that, over the short term, there is negative compensation for volatility risk premiums, akin to the equity or currency markets. Over the long term, the signs of compensation change and regression tests show the possibility that the volatility risk premium in the interest rate market can be different from those in other asset markets. However, this interpretation entails an overlapping data problem that is not easy to overcome especially for the long term DHG data. The difference in interest rate market may be due to the fact that the interest rate swap market is different from the equity or currency markets in that it is more driven by financial institutions and option traders than by individuals or directional traders.

1. Introduction

Why are traders willing to buy expensive options? According to existing research on the equity option and currency option markets, option buyers usually lose money because these markets compensate negatively for volatility risk. This implies that people tend to want to buy options or volatility rather than to sell. This study attempts to examine whether this phenomenon occurs in the interest swap markets as well.

Regardless of their theories, practitioners trade options involving swaps (swaptions) and hedge their delta risks dynamically using interest rate instruments, usually interest rate swaps. In other words, they make or lose money by using dynamic delta hedging strategies. Academics refer to these gains or losses as delta-hedged gains (DHGs). If the volatility of swaptions is constant, these gains should average zero, but if it is stochastic and is compensated for by markets, the average will be different. This aspect is also analyzed in this study.

Roughly 20 years have passed since Heston (1993) suggested the closed form formula for option pricing where volatility follows the square root process. Changes in volatility are generally accepted in interest rate markets; therefore, stochastic volatility models for interest rate markets have been proposed in recent times. For example, Hagan, Kumar, Lesniewski, and Woodward (2002) proposed the stochastic alpha beta rho (SABR) model, while Wu and Zhang (2006) and Trolle and Schwartz (2009) suggested the stochastic bond price and volatility model.

Apart from these modeling approaches, there was no empirical research on volatility risk premiums in interest rate markets prior to Fornari (2010). He studied volatility risk premiums in interest rate swap markets by using data for three currencies, USD, EUR, and GBP, from 1998 to 2006. He used a method based on the GARCH model to estimate realized volatility and volatility under a physical measure. To estimate volatility under a risk-neutral measure, he used swaption-implied volatilities. He regarded the spread between the two volatilities as compensation for volatility risk. He concluded that interest rate volatility leads to negative compensation for volatility risk, which is in line with other studies focusing on different asset classes. He also finds that the process of selecting a particular model to calculate realized volatility can be arbitrary. To avoid this issue, Bollerslev, Tauchen, and Zhou (2009) used a model-free approach to calculate volatility spread. They applied their technique to stock markets because they required high-frequency intraday data that is seldom found in over-the-counter markets such as interest rate swap and swaption markets.

Few existing studies have explored volatility risk premiums in interest rate markets. As a result, the methodology could be restricted. In order to address this gap, this study analyzes the interest rate swap market and uses the Bakshi and Kapadia (2003); (hereafter BK) method to estimate volatility risk premiums. Therefore our contribution is, on the

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one hand, showing that the BK method can be applied to the market where the forward measure or annuity measure should be used. This procedure is also necessary from the fact that the interest rate market is different from the equity and currency markets in practical and theoretical ways. Applying this to extended market data of interest rate swap covering the financial crisis of 2008, on the other hand, it could contribute to the understanding of volatility risk premium in interest rate markets.

The rest of this study is organized as follows. Section 2 introduces the instrument and market examined in this study. Section 3 demonstrates that the BK method is applicable to the interest rate swap markets and also explains the methodology involved. Section 4 presents the data used and the empirical findings of this study. Section 5 explains the robustness of our analysis. Finally, Section 6 provides the conclusion.

2. Instruments and markets

2.1. Interest rate swap

A number of terms related to interest rate markets that are used in this study are described as follows. Year fraction $\tau(t,T)$ is the number of days between date t and date T divided by the number of days in a year. Consider an investment of one unit of currency at date t and returns of R units at date T; the simply-compounded spot interest rates $L(t,T)$ for this investment can be given as:

$$L(t,T)=\frac{R-1}{\tau(t,T)}. \quad (1)$$

An interest-rate swap (hereafter IRS or swap) is a contract that exchanges payments between two differently indexed legs on a prespecified set of dates $T_{\alpha+1}, \ldots, T_\beta$. On the payment date T_i ($i = \alpha + 1,\ldots,\beta$), the fixed leg pays out the amount:

$$N\tau_i K, \quad (2)$$

corresponding to the fixed interest rate K, the nominal amount N, and a year fraction $\tau_i = \tau(T_{i-1}, T_i)$ between T_{i-1} and T_i. On the same date, the floating leg pays out the amount:

$$N\tau_i L(T_{i-1}, T_i) \quad (3)$$

corresponding to the interest rate $L(T_{i-1}, T_i)$, reset at the previous payment date T_{i-1} for the maturity based on the current payment date T_i. Note that the floating-leg rate resets on dates $T_\alpha, \ldots, T_{\beta-1}$ and pays out on dates $T_{\alpha+1}, \ldots, T_\beta$. If the fixed leg is paid and the floating leg is received, the IRS is called a payer IRS (swap). If the floating leg is paid and the fixed leg is received, the IRS is called a receiver IRS (swap).

The spot IRS or spot swap is one whose first reset date is on the usual spot date. For example, in the USD swap market, the spot date is two business days after the trade date. If a swap is traded for which the first reset date is later than the spot date, it is called a forward IRS or forward swap.

The forward swap rate is the fixed rate of the relevant swap contract that makes the value of the forward swap zero. When discount factors exist, the forward swap rate can be easily calculated using its definition. The time t forward swap rate $S_{\alpha\beta}(t)$ with the first rest date t_α and the last payment date t_β is

$$S_{\alpha\beta}(t)=\frac{B(t,t_\alpha)-B\left(t,t_\beta\right)}{\sum_{i=\alpha+1}^{\beta}B(t,t_i)\tau_i} \quad (4)$$

where $B(t,T)$ is the time t price of the zero coupon bond that pays the unit currency at T.

2.2. Swaptions and their market conventions

An option on a swap is referred to as a swaption. There are two types of swaptions—European payer swaption and European receiver swaption. The former is an option giving the right but not the obligation to enter into a payer IRS at the swaption's maturity date. The latter is an option giving the right to enter into a receiver IRS at the swaption's maturity date. The length of the underlying swap $T_\beta - T_\alpha$ is called the tenor of the swaption.

Like traders in the currency options market, swaption market traders quote prices based on volatility and calculate option premiums by inputting the volatility into the following Black (1976) model:

$$PS^{Black}(0,\overline{T},\tau,N,K,\sigma)=NBl\left(K,S_{\alpha\beta}(0),\sigma\sqrt{T},1\right)\sum_{i=\alpha+1}^{\beta}\tau_i B(0,T_i) \quad (5)$$

$$RS^{Black}(0,\overline{T},\tau,N,K,\sigma)=NBl\left(K,S_{\alpha\beta}(0),\sigma\sqrt{T},-1\right)\sum_{i=\alpha+1}^{\beta}\tau_i B(0,T_i) \quad (6)$$

where:

PS^{Black}	Payer swaption price using the Black model
RS^{Black}	Receiver swaption price using the Black model
N	Notional amount
$Bl(K,F,v,w)=Fw\Phi(wd_1)-Kw\Phi(wd_2)$	Non-discounted Black price
$d_1=\frac{\ln(F/K)+v^2/2}{v},\ d_2=\frac{\ln(F/K)-v^2/2}{v}$	
ω	1 if payer swaption, otherwise −1
Φ(x)	Cumulative standard normal density function
$\overline{T}=\{T_\alpha,\ldots,T_\beta\}$	Set of dates (reset dates and payment dates)
$\tau=\{\tau_{\alpha+1},\ldots,\tau_\beta\}$	Set of year fractions
B(0,T)	Present value of discount bonds that pay one unit of currency on date T
T	Option expiry
T_α	First reset date, the effective date of swap
T_β	Last payment date
$S_{\alpha,\beta}(t)$	Forward swap rate at time t, for a swap with first reset date t_α and last payment date t_β.
σ	Volatility of forward swap rates
K	Strike of swaption

This volatility σ is called Black volatility.

The pricing formula presented above can be applied by assuming that the forward swap rate dynamics are as in Eq. (7), and by using the results of the lognormal forward-swap model (LSM or swap market model) (Brigo & Mercurio, 2007; Jamshidian, 1997) where the forward swap rates are martingales.

$$\frac{dS_t}{S_t}=\mu dt+\sigma_t dW_t^P \quad (7)$$

where S_t is the abbreviation for the forward swap rate $S_{\alpha,\beta}(t)$. W_t is the standard Wienner process, while superscript P denotes the dynamics defined by a physical measure.

In recent times, practitioners have often quoted swaption prices by using the premium itself rather than volatility. However, the premium can be easily converted into Black volatility and information providers such as Reuters and Bloomberg provide both premium and Black volatility. As such, recent changes in quotation methods do not affect this research.

3. Delta-hedged gain and interest rate swap markets

This section provides an overview of an interest rate model with stochastic volatility, and reviews the meaning of DHG. In this section, forward swap rate dynamics are derived using various measures, such