A non-linear dynamic model of the variance risk premium

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ABSTRACT

We propose a new class of non-linear diffusion processes for modeling financial markets data. Our non-linear diffusions are obtained as transformations of affine processes. We show that asset-pricing and estimation is possible and likelihood estimation is straightforward. We estimate a non-linear diffusion model for the VIX index under both the objective measure and the risk-neutral measure where the latter is obtained from futures prices. We find evidence of significant non-linearity under both measures. We define the difference between the \( P \) and \( Q \) drift as a measure of the variance risk premium and show that it has strong predictive power for stock returns.

1. Introduction

Non-linear diffusion processes are appealing tools for modeling financial prices. Non-linearities in the drift of a diffusion can imply that a process will mean-revert quickly from high levels. For example, Aït-Sahalia (1996), Stanton (1997), and Bandi (2002) among others, find that interest rates exhibit such non-linearities. Bandi and Reno (2012) study non-parametric jump diffusions for volatility. There is also considerable evidence that many economic processes do not exhibit exponentially decaying autocorrelation functions, as implied by linear models.

Despite the appealing characteristics of non-linear models in representing economic data such as nominal interest rates and volatility, their use in pricing applications is limited. The primary reason for this is that general, unrestricted non-linear models, while providing excellent fit to data, are typically difficult to use in asset pricing applications as asset prices are only available through computationally intensive methods. Moreover, unrestricted non-linear processes are difficult to constrain to a subspace of \( R^d \) which is sometimes required (i.e. for positivity and or stationarity).

This paper makes two contributions. First, we propose a method for deriving a class of non-linear diffusion processes that have the desirable properties that allow us to use them in modeling financial market prices. Our class of such diffusion processes is derived as non-linear transformations of basic affine processes. The originating process, say \( x_t \), can be an OI or CIR process. The transformed process, \( y_t = f(x_t) \), has the following three desirable properties

1. It can be written as a diffusion
\[
dy_t = \mu_y(y_t)dt + \sigma_y(y_t)dB_t
\]
where \( \mu_y \) and \( \sigma_y \) are non-linear drift and diffusion functions.
2. The transition density \( p(y_{t+1} \mid y_t) \) is known.
3. If \( \mu(y_t, \varphi_Q) \) and \( \sigma(y_t, \varphi_Q) \) represent the drift and diffusion under a risk neutral measure \( Q \), then the price of a contingent claim with payoff \( G(y_{t+1}) \) is available up to an integral equation.

Second, we propose a non-linear model to describe the VIX index and the variance risk premium. The VIX index is interpretable as an approximate market-implied estimate of the one-month ahead conditional volatility of the S&P 500 index conditional log-return under the risk neutral measure. So if \( R_{1:22} \) denotes the cumulative logarithmic twenty-two business day return on the S&P 500 index, the VIX index is a market implied estimate of \( VIX^2 = Var^0(R_{1:22}) \). In our empirical application, we model the VIX (or squared VIX) as a non-linear diffusion through our transformation such that \( y_t = VIX^2 \). We report MLE and GMM estimates of the non-linear diffusion specification for the squared VIX.

We also study the variance risk premium. This risk premium, referred to as the variance (or volatility) risk premium, is defined as a difference between risk-neutral and objective measure volatility.

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Specifically, the standard definition (see for example Bollerslev et al., 2009)

\[ V_{t+1} = \operatorname{Var}_t^Q (\ln S_{t+1}) - \operatorname{Var}_t (\ln S_{t+1}) \]  

(2)

The variance risk premium is almost always positive, and is of a magnitude that has some important financial market implications, including “over-pricing” of S&P 500 put and call options.\(^1\)

Our measure of the variance risk premium is not based directly on estimates of the two variances in (2), as is the common approach in the extant literature. Rather, our estimator is based on the difference in the drift for the squared VIX under the objective and risk-neutral measures. We show that our definition of the market-risk premium of VIX returns is equivalent to the definition in the extant literature which is based on Eq. (2).

There is a large literature on asset pricing using affine processes (see Singleton, 2006) for an extensive review). There exist few special cases offer analytical transition densities. The conditional characteristic function \( \mathbb{E}_t (\varphi(X_T)) = \mathbb{E}_t \left( \varphi(X_T) \right) \) where \( \varphi \) is a functional (non-linear) transform of a diffusion process, \( X_t \), with known properties. Specifically, let \( \Omega = (\Omega, \mathcal{F}, \mathbb{P}) \) denote a probability space with an information filtration \( (\mathcal{F}_t) \). The idea is to consider a functional (non-linear) transform of a diffusion process, \( x_t \), with known properties. Specifically, we have

Assumption 1. We assume that \( x_t \) is a stationary Markov process with domain \( D \subset \mathbb{R}^n \),

\[ dx_t = \mu (x_t, \theta) dt + \sigma (x_t, \theta) dB_t, \]

(4)

where \( \mu : D \times \mathbb{R}^k \to \mathbb{R}^n \) and \( \sigma : D \times \mathbb{R}^k \to \mathbb{R}^n \) are the drift and diffusion functions which we assume to be parameterized by a \( k \) dimensional parameter \( \theta \). Let \( \mu_i (x_t) \) and \( \sigma_{ij} (x_t) \) denote the \( i \)th element of the drift function and \((i, j)\)th element of diffusion function, respectively.

Assumption 2. The conditional characteristic function \( \phi_t (u) = \mathbb{E}_t \exp (i u x_{t+1}) \) for \( u \in \mathbb{C}^n \) can be computed explicitly.

Assumption 3. The transition density \( p(x_{t+1} | x_t, \theta) \) can be computed explicitly.

By “computed explicitly” we mean that the functions can be computed either analytically or through numerical methods that are computationally fast. Assumption 2 is needed for asset pricing purposes while Assumption 3 facilitates likelihood based inference. A candidate for \( x \) is the general affine class of diffusions considered by Duffie et al. (2000). The affine class generally offers analytical tractability of the generating functions (up to ODE’s) and special cases offer analytical transition densities.
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