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Robust portfolio rules and detection-error probabilities for a mean-reverting risk premium

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Abstract

I analyze the optimal intertemporal portfolio problem of an investor who worries about model misspecification and insists on robust decision rules when facing a mean-reverting risk premium. The desire for robustness lowers the total equity share, but increases the proportion of the intertemporal hedging demand. I present a methodology for calculation of detection-error probabilities, which is based on Fourier inversion of the conditional characteristic functions of the Radon–Nikodym derivatives. The quantitative effect of robustness is more modest than in i.i.d. settings, because model discrimination between the benchmark and the worst-case alternative model is easier, as indicated by the detection-error probabilities.

JEL classification: G11; G12

Keywords: Portfolio choice; Robustness; Model uncertainty; Intertemporal hedging; Detection-error probability

The seminal work of Merton [57] shows that dynamic portfolio allocation is different from single-period investing: non-myopic long-term investors consider not only the present risk-return trade-off they face, but also take into account future investment opportunities. In particular, these investors optimally deviate from the instantaneously mean-variance efficient portfolio in order to hedge against predictable changes in future risk-return trade-offs. For instance, an asset that tends to pay off well when future investment opportunities are expected to worsen provides a hedge against this deterioration of future investment prospects. Relatively conservative investors value this hedge and will have a positive hedging demand for the asset: they hold more of it than would be justified by the current risk-return trade-off only. Empirically, this is the case for equity when accounting for mean reversion in the risk premium (see e.g. [6,16]).

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Intertemporal hedging of changes in a mean-reverting risk premium has a number of fundamental implications for portfolio choice. First, the demand for equity increases substantially for investors that are more risk-averse than log (the empirically more relevant case). These moderately risk-averse investors are often predicted to optimally hold highly levered positions when incorporating hedging. [1] Second, hedging makes the optimal equity allocation time-varying and horizon-dependent. Indeed, the result of Merton [56] and Samuelson [60] of constant optimal portfolio shares depends crucially on the assumption of i.i.d. returns. Mean reversion and hedging can therefore provide a rationalization for the horizon- or age-dependent portfolio rules that are typically advocated by financial planners and popular investment books. Finally, hedging induces a form of (low-frequency) market-timing.

Intertemporal hedging is fundamentally driven by time-variation in investment opportunities, usually modeled in the form of predictable time-variation in expected returns. While there is a large empirical literature documenting the ability of (for instance) the dividend yield to predict expected equity returns (see e.g. [23, Chapter 20]), there is some controversy surrounding these results. [2] Because of this active debate, it is fair to say that the process describing the dynamics of investment opportunities is subject to a substantial degree of uncertainty. This uncertainty makes it difficult to justify treating the model that governs the return dynamics as known and fixed. Nevertheless, this is what is typically done in the recent literature that operationalizes Merton's idea and that solves explicitly for the hedging demands. The bulk of this work [3] postulates a particular model for the return dynamics, obtains point estimates for its parameters and subsequently assumes that the investor relies on the estimated model with full confidence and without any concerns about the validity of the model itself.

This paper, instead, explicitly incorporates model uncertainty into the analysis and solves the dynamic portfolio problem for an investor worrying about model uncertainty in addition to market risk. The concern about model uncertainty leads the investor to insist on portfolio rules that are robust to model misspecification: robust portfolio rules are designed to work well not only if the underlying model describing the return dynamics is specified correctly, but to also perform reasonably well if there is some form of model misspecification. Formally, this is achieved by solving the dynamic portfolio problem with stochastic investment opportunities using techniques of robust control, as introduced in economics by Hansen and Sargent [37]. A robust decision-maker entertains a family of alternative models (for the return dynamics) around the benchmark model, which is deemed useful but subject to misspecification, and guards against a worst-case alternative model that is difficult to distinguish statistically from the benchmark model (according to a log-likelihood ratio or relative entropy criterion, weighted by a preference parameter that measures

[1] For instance, the results in Campbell et al. [14] indicate that incorporating intertemporal hedging in the portfolio problem of an infinitely-lived investor with coefficient of relative risk aversion equal to 4, increases the mean fraction of wealth invested in equities from 53% (the myopic demand) to 180% (the optimal demand).

[2] A number of papers criticize the statistical techniques used to establish the predictability. When correcting the inference methods, different results emerge. See for instance Ang and Bekaert [3], Bossaerts and Hillion [8], Campbell and Yogo [19], Ferson et al. [32], Goetzmann and Jorion [35], Goyal and Welch [36], Hodrick [43], Lewellen [50], Lo and MacKinlay [53], Richardson and Stock [59], Stambaugh [64], and Torous et al. [66]. Other work suggests that the data generating process for equity returns is subject to structural breaks [58,44], and that the predictability of stock returns may also have changed structurally over time [45,36,68].

[3] See for instance Balduzzi and Lynch [5], Barberis [6], Brandt [9], Campbell and Viceira [16], Kim and Omberg [46], Lynch [54], Schroder and Skiadas [61] and Wachter [69] for stochastic excess returns; Brennan and Xia [12] and Campbell and Viceira [17] for stochastic interest rates; and Brennan et al. [11], Campbell et al. [15] and Liu [51], who analyze both. See Campbell and Viceira [18] for a more exhaustive bibliography.