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Solving the incomplete markets model with aggregate uncertainty using the Krusell–Smith algorithm

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ABSTRACT

This paper studies the properties of the solution to the heterogeneous agents model in Den Haan et al. [2009. Computational suite of models with heterogeneous agents: incomplete markets and aggregate uncertainty. Journal of Economic Dynamics and Control, this issue]. To solve for the individual policy rules, we use an Euler-equation method iterating on a grid of pre-specified points. To compute the aggregate law of motion, we use the stochastic-simulation approach of Krusell and Smith [1998. Income and wealth heterogeneity in the macroeconomy. Journal of Political Economy 106, 868–896]. We also compare the stochastic- and non-stochastic-simulation versions of the Krusell–Smith algorithm, and we find that the two versions are similar in terms of their speed and accuracy.

1. Introduction

This paper studies the properties of the solution to the incomplete markets model with aggregate uncertainty in Den Haan et al. (2009). Our solution method consists of two interconnected steps: the first is to solve the individual problem for a given aggregate behavior of the economy and the second is to compute the aggregate law of motion for the given individual policy rules. We iterate on these two steps until we find a fixed point at which the individual and aggregate policy rules are mutually consistent.

Step one is straightforward: the individual problem is the typical capital-accumulation problem with an occasionally binding borrowing constraint, and it can be solved by the standard numerical methods. We solve the individual problem by using a grid-based Euler-equation algorithm similar to that in Maliar and Maliar (2005, 2006). We extend Maliar and Maliar's (2005, 2006) algorithm by incorporating a simple polynomial rule for constructing the grid, which allows us to vary the concentration of capital grid points on different parts of the domain, thus increasing the accuracy of approximation on non-linear parts of the policy rules. Our algorithm is also similar to the grid-based Euler-equation method used by Baxter et al. (1990) for solving the standard one-sector growth model. Furthermore, our algorithm is related to the parameterized expectations algorithm used in Den Haan and Marcet (1990), Den Haan (1997), Christiano and

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Fisher (2000), Maliar and Maliar (2003b), and Algan et al. (2008). However, the above papers parameterize an expectation term in the Euler equation and use a polynomial approximation, whereas we parameterize a capital function and compute a solution on a grid of pre-specified points.[1]

Step two is non-trivial. Decisions of each heterogeneous agent depend on the interest rate and wage rate, which in turn depend on the aggregate capital stock. Since the aggregate capital stock is determined by capital holdings of all heterogeneous agents, the whole capital distribution becomes a state variable.[2] With a continuum of agents, this distribution is a function, and therefore, it cannot be used as an argument of the individual policy rules. To deal with this problem, Krusell and Smith (1998) propose to summarize the capital distribution by a discrete and finite set of moments.[3] They solve the individual problem by using value iteration, and they compute the aggregate law of motion by simulating a panel for a large finite number of agents and by running regressions on the simulated data. In this paper, we follow the stochastic-simulation approach of Krusell and Smith (1998). Consequently, our solution procedure is a variant of the Krusell–Smith algorithm, specifically one in which the individual problem is solved by an Euler-equation method instead of Krusell and Smith's (1998) value function iteration. Our computer programs are written in MATLAB in an instructive manner and are provided on the JEDC web site (see the web pages of the authors for updated versions of the program).

An important advantage of the stochastic-simulation Krusell–Smith algorithm is that it is simple, intuitive and easy to program. As Algan et al. (2008) show, however, stochastic-simulation methods have two potential shortcomings. First, the introduction of stochastic simulations produces sampling noise, which makes the policy rules to depend on a specific random draw. Second, the simulated endogenous data are clustered around the mean, which implies that the accuracy of the approximation on the tails is low. They argue that replacing a stochastic simulation with a non-stochastic one can enhance the accuracy and speed of the algorithm. Therefore, it is of interest to assess the accuracy of the stochastic-simulation version of the Krusell–Smith algorithm and to compare it with a non-stochastic-simulation version.

We find that, despite the above shortcomings, the stochastic-simulation Krusell–Smith method produces sufficiently accurate solutions.[4] This is true even under our relatively small panel of 10,000 agents and relatively short simulation length of 1,100 periods. For example, in an accuracy test where the model was simulated on a random realization of shocks of 10,000 periods, the average and maximum errors in our aggregate capital series were 0.050% and 0.156%, respectively. Furthermore, we consider a non-stochastic-simulation Krusell–Smith algorithm where simulations are performed on a grid of pre-specified points, as is described in the appendix in Den Haan (2009).[5] We find that the benchmark stochastic-simulation version of the Krusell–Smith algorithm with a panel of 10,000 agents has approximately the same cost as the non-stochastic-simulation version with a grid of 1,000 points and produces solutions of comparable (or even higher) accuracy. Thus, in our case, the introduction of non-stochastic simulation does not lead to substantial improvements.

2. The individual problem

In this section, we describe an Euler-equation algorithm for finding a solution to the individual problem described in Den Haan et al. (2009). This is the standard capital-accumulation problem with an occasionally binding borrowing constraint. The Euler equation, the budget constraint, the borrowing constraint and the Kuhn–Tucker conditions, respectively, are

$$c^{-\gamma} - h = \beta E\{(c')^{-\gamma}(1-\delta+r')\}, \tag{1}$$

$$k' = (1-\tau)w\bar{l}\varepsilon + \mu w(1-\varepsilon) + (1-\delta+r)k - c, \tag{2}$$

$$k' \geq 0, \tag{3}$$

$$h \geq 0, \quad hk' = 0, \tag{4}$$

where variables without and with primes refer to the current and future periods, respectively (we omit the individual superscripts for the sake of notational convenience). Here, c is consumption; k is capital; ε is an idiosyncratic shock that determines an employment status, with $\varepsilon = 1$ and $\varepsilon = 0$ representing the employed and unemployed states, respectively; h

[1] For a general discussion of the Euler-equation methods, see Judd (1998).

[2] Under the assumption of complete markets, the aggregate behavior of a similar heterogeneous-agent economy with idiosyncratic and aggregate uncertainty can be described by a one-consumer model; see Maliar and Maliar (2003a) for this aggregation result. In this special case, the state space does not include the whole capital distribution but only its mean.

[3] Den Haan (1997) proposes an alternative approach for dealing with this problem, namely, to parameterize the cross-sectional distribution with a polynomial.

[4] An exception is very large errors produced by our method in a dynamic Euler-equation accuracy test, see Table 14 in Den Haan (2009). A typo in our program is responsible for these large errors. After we corrected the typo, the errors became considerably lower, namely, in Table 14, the capital (scaled) average and maximum errors should be equal to 0.0319% and 0.0926%, respectively, and the consumption average and maximum errors should be equal to 0.0091% and 0.4360%, respectively.

[5] This non-stochastic-simulation procedure is close to the one considered in Rios-Rull (1997). A different non-stochastic-simulation procedure is proposed by Young (2009), who was the first to combine the Krusell–Smith algorithm with non-stochastic simulation. Algan et al. (2008) perform a comparison of Rios-Rull's (1997) and Young's (2009) and their own procedures.