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Price-level volatility and welfare in incomplete markets with sunspots

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ABSTRACT

In an economy with incomplete financial markets as described by Cass (1989), there is typically a continuum of equilibria driven by sunspots. In some cases, there is no Pareto ranking among the different sunspot equilibria. However, this paper shows that a sunspot equilibrium with lower price-volatility is superior in economic welfare to one with higher price-volatility based on a compensation test of balanced tax-transfer plans. Specifically, I start with a non-singular benchmark equilibrium. For any nearby equilibrium prices with smaller volatility, there exists a small redistribution of first period endowments that achieves an equilibrium with the same price-volatility but is yet Pareto-superior to the benchmark equilibrium. Such a Pareto-improving redistribution does not exist for the nearby equilibrium with higher price-volatility.

1. Introduction

Sunspots provide explanations of excess volatility of both price levels and allocations.[1] Cass (1989, 1992) showed that when markets are incomplete, there is typically a continuum of sunspot equilibria. He also implicitly showed that generically in endowment, there is a finite number of equilibria for any fixed ratio of price levels in a two-state model.[2] Even if the equilibria exhibit different levels of price volatility,[3] they are not, in general, Pareto-ordered by price volatility levels. That is initially shown in the pioneering work by Goenka and Préchac (2006).[4] Kajii (2007) further showed that there is a general-equilibrium effect that benefits borrowers or harms lenders, while in partial equilibrium, the excess volatility inevitably harms each agent. Thus, in general, Pareto criterion cannot rank sunspot equilibria of different levels of price volatility, because the asymmetric general equilibrium effect cannot be compensated. However, it is still intuitive that a sunspot equilibrium of lower volatility is higher in economic welfare because the economy suffers less from purely extrinsic risks.

Therefore, this paper considers a different criterion that allows ex-ante transfers, similar in spirit to the Kaldor criterion, to argue that a sunspot equilibrium with lower volatility is indeed superior. I mainly focus on local analysis to take advantage of differential analysis and the indeterminacy of sunspot equilibria. Specifically, this paper starts with a non-singular benchmark equilibrium. For any nearby equilibrium with smaller price-volatility, there exists a small redistribution of endowments such that there exists an equilibrium with the same smaller price-volatility which is Pareto superior to the benchmark equilibrium. In addition, this paper shows that it is impossible to find a redistribution in which a high volatility equilibrium of the resulting economy Pareto-improves a low volatility equilibrium of the original economy. For the local welfare analysis, this paper adopts Geanakoplos and Mas-Colell's (1989) idea that an economy with nominal assets can be parameterized by economies with real assets. In this paper, the relative standard deviation of the equivalent real assets is used to measure price-level volatility.

Mathematically, our analysis develops the idea of Pareto-improving tax-transfer plans in incomplete markets, which is originally from Cittana, Kajii, and Villanacci (CKV, 1998). CKV showed the existence of welfare-improving redistribution plans in an economy with numeraire assets and fixed asset returns. CKV assumes that there are multiple commodity goods in each spot market so that there exists a Pareto-improving redistribution even with fixed asset payoffs. However, under the assumption of one commodity in an economy with real financial assets, it is impossible to achieve Pareto improvement through a redistribution policy because the equilibrium allocations are constrained Pareto-optimal by Diamond (1967). The model in this paper has only one commodity in each spot market. Thus, any equilibrium is constrained optimal so that it is not possible to improve welfare through a redistribution policy if the asset real returns are fixed. Therefore,

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1 See Cass and Shell (1983) and Shell (1987, 2008).

2 See p. 686 for the proof of Proposition 2.

3 In this paper, the measure of price volatility is the ratio of two price levels, equivalently, the relative standard deviation of the price level in a two-state model. See Section 3.

4 Also, see Cozzi et al. (2012).

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changes of asset returns, that is, changes of price-level volatility, will be considered with redistribution plans to achieve Pareto improvement in the local analysis.

The main difficulty in the extension of the local result to the global (which compares two equilibria with distinct levels of price volatilities) is the "singularity" issue: the possibility that there is a singular point between two equilibria from the two distinct volatility levels. However, as shown in Goenka and Préchac (2006), there is a class of economies where non-singularity is guaranteed for any positive endowment and any volatility level, such as log-linear preferences. This paper provides two numerical examples where the global analysis can be applied. In the global analysis, this paper compares any two economies with the same economic fundamentals, but with different price volatilities. Specifically, it demonstrates that there are balanced lump-sum tax plans that would allow an economy with lower volatility to be Pareto-superior to one with higher volatility, in a class of economy where there is no potential singular point.

This finding has important policy implications. Several studies have suggested stabilizing policies to eliminate the effect of sunspots on incomplete markets. Four dominant policies have been proven: (a) the introduction of new types of nominal securities,[5] (b) the introduction of as many real securities as the number of goods in each state,[6] (c) the indexation of nominal bonds in terms of price levels[7] and (d) the introduction of options.[8] These policies immunize the economy from sunspot effects; consequently, the outcomes of the equilibria are Pareto efficient. However, efficiency does not imply that all consumers are better off at non-sunspot equilibria. The recent sunspot literature has shown that many consumers can actually benefit from sunspots.[9] This means that the government can fail to gain consensus in adopting stabilizing policies. Nevertheless, this paper shows that, if balanced lump-sum tax plans are allowed, consumer consensus on sunspot-stabilizing policies can be achieved in a class of economies without any potential singular point.

The remainder of this paper is organized as follows. In Section 2, I introduce the general setting of the model. Section 3 presents the main results in Cass (1989) and defines price-level volatility in terms of the relative standard deviation. Section 4 presents the local welfare analysis investigating the welfare change caused by changes of price volatility and tax-transfer plans. The main result of this paper is shown in Section 5. In Section 6, the local analysis is extended into the global one in a class of economies without potential singular point. Finally, concluding remarks are presented in Section 7.

2. The model

There are two periods, today and tomorrow, labelled by the superscripts $t = 0, 1$. At date 1, there are two states, $s = \alpha, \beta$ with positive probabilities $0 < \pi^{\alpha} < 1$ and $\pi^{\beta} = 1 - \pi^{\alpha}$, respectively. There are H consumers, labelled by the subscripts $h \in H = \{1, 2, \ldots, H\}$.[10]

Consumer h's consumption allocation is $x_h = \left(x_h^0, x_h^{1\alpha}, x_h^{1\beta}\right) \in X = \mathbb{R}^3_{++}$ corresponding to price $p = \left(p^0, p^{1\alpha}, p^{1\beta}\right) \gg 0$. His endowment is $e_h = \left(e_h^0, e_h^{1\alpha}, e_h^{1\beta}\right) \in X$, where $e_h^{1\alpha} = e_h^{1\beta} = e_h^1$. Consumer h's preference is

$$u_h(x_h) = u_h\left(x_h^0, x_h^{1\alpha}, x_h^{1\beta}\right) = \pi^{\alpha} v_h\left(x_h^0, x_h^{1\alpha}\right) + \pi^{\beta} v_h\left(x_h^0, x_h^{1\beta}\right)$$

where the sub-utility function $v_h(\cdot)$ is strictly increasing, strictly concave, and twice-continuously differentiable. Throughout the paper, I assume that there is an incentive for at least two of the consumers to trade, i.e., for $h, h' \in H$,

$$\frac{\partial v_h\left(e_h^0, e_h^1\right)}{\partial x^1} \bigg/ \frac{\partial v_h\left(e_h^0, e_h^1\right)}{\partial x^0} \neq \frac{\partial v_{h'}\left(e_{h'}^0, e_{h'}^1\right)}{\partial x^1} \bigg/ \frac{\partial v_{h'}\left(e_{h'}^0, e_{h'}^1\right)}{\partial x^0}.$$

This condition implies that the initial endowment is not Pareto-efficient.

In a monetary market, there is only one financial instrument (money). m_h denotes consumer h's money holdings. The monetary equilibrium is defined as follows: there are some positive spot prices $p \gg 0$ and associated money holdings m such that each household chooses (x_h, m_h) in the optimization problem, denoted as (PA).

$$\max\ u_h\left(x_h^0, x_h^{1\alpha}, x_h^{1\beta}\right)$$

$$\text{subject to} \begin{cases} p^0 x_h^0 + m_h \leq p^0 e_h^0 \\ p^{1\alpha} x_h^{1\alpha} \leq p^{1\alpha} e_h^1 + m_h \\ p^{1\beta} x_h^{1\beta} \leq p^{1\beta} e_h^1 + m_h \end{cases} \tag{PA}$$

and $x_h \in X$

and each market clears,

$$\sum_h x_h^0 = \sum_h e_h^0,$$

$$\sum_h x_h^{1\alpha} = \sum_h x_h^{1\beta} = \sum_h e_h^1$$

$$\sum_h m_h = 0.$$

3. The equilibrium

The economy has only one financial asset (money) but two states, α and β, at date 1. Therefore, the equilibrium set has one degree of real indeterminacy.[11] The real returns of money, denoted as $\left(R^{\alpha}, R^{\beta}\right)$ have an inverse relationship with the price levels $\left(p^{1\alpha}, p^{1\beta}\right)$, i.e., $\frac{R^{\beta}}{R^{\alpha}} = \frac{p^{1\alpha}}{p^{1\beta}}$, because the ratio of two prices $p^{1\alpha}$ and $p^{1\beta}$ determines the ratio between excess demand in each state: $x^{1\alpha} - e^1$ and $x^{1\beta} - e^1$. This relationship is derived from the budget constraint at date 1:

$$\begin{cases} p^{1\alpha} x_h^{1\alpha} = p^{1\alpha} e^1 + m_h \\ p^{1\beta} x_h^{1\beta} = p^{1\beta} e^1 + m_h \end{cases} \Rightarrow \frac{p^{1\alpha}}{p^{1\beta}} = \frac{x_h^{1\beta} - e^1}{x_h^{1\alpha} - e^1} \equiv \left(\frac{R^{\beta}}{R^{\alpha}}\right).$$

Fig. 1(a) represents the excess-demand domains of both lenders and borrowers.[12] Once the return-line is determined, there will be a unique or finitely many equilibria point(s) for both the lender ($m_h > 0$) and the borrower ($m_h < 0$) on the line.[13] The lender's allocation point is located in the northeast area, while the bor-

5 See Cass and Shell (1983, Proposition 3) and Balasko (1983, Theorem 1).

6 See Mas-Colell (1992).

7 See Goenka and Préchac (2006).

8 See Antinolfi and Keister (1998) and Kajii (1997).

9 See Bhattacharya et al. (1998), Goenka and Préchac (2006), Kajii (2007) and Cozzi et al. (2012).

10 There are two differences between Cass's model and the model here: (1) Cass allows only two households, but the model here allows many households, and (2) Cass distinguishes the asset return in the α-state with that in β state and denoted them as r^{α} and r^{β}.

11 Cass (1989, 1991, 1992) and Siconolfi (1991) have demonstrated that in a general model of sunspots with incomplete markets, the set of equilibrium allocations takes on a continuum. Manuelli and James (1992) also showed that in an overlapping generation model with incomplete markets, a continuum of sunspot equilibria can be interpreted as the limiting case of economies overreacting to small shocks to fundamentals.

12 Fig. 1 should be three-dimensional, including the excess demand of x^0, but we can imagine that the three-dimensional figure is projected onto a two-dimensional space.

13 In the monetary market, the asset buyer and seller can be translated into the money lender and the money borrower, respectively.