Journal of Mathematical Economics 50 (2014) 119–127

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

The relation between sunspot effects and multiplicity in incomplete markets models with numeraire assets☆

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ARTICLE INFO

Article history:
Received 19 July 2012
Received in revised form
15 July 2013
Accepted 27 October 2013
Available online 14 November 2013

Keywords:
Sunspots
Extrinsic uncertainty
Numeraire assets
Incomplete markets
Randomization

ABSTRACT

This paper considers the necessity and sufficiency of multiple certainty equilibria for sunspot effects, and shows that neither implication is valid. This claim is made for models with incomplete markets and numeraire assets. First, I prove that a multiplicity of certainty equilibria is neither necessary nor sufficient for sunspot effects by way of two counter-examples. Second, I verify over an entire subset of economies that equilibrium with sunspot effects can never be characterized as a randomization over multiple certainty equilibria.

1. Introduction

The idea of sunspots has attracted considerable attention from economists, because sunspots are a formal representation of seemingly irrational behavior exhibited in financial markets. In these markets, traders respond to irrelevant information that has no bearing on fundamentals. These responses, if adopted as the market psychology, result in self-fulfilling optimal actions by traders. In terms of economic modeling, sunspots are realizations of extrinsic uncertainty, i.e., uncertainty that does not impact the fundamentals of the economy (endowments and preferences of households and asset payouts). As was shown thirty years ago (Cass and Shell, 1983; Balasko, 1983; Azariadis, 1981), even when the tenet of rational expectations is maintained, sunspots can affect the real equilibrium variables.

Since the founding articles on the topic (Cass and Shell, 1983; Balasko, 1983; Azariadis, 1981), the origin of sunspot effects has been a source of confusion for economists. Initially, one of the leading explanations for sunspot effects is that they occur exclusively in economies with multiple certainty equilibria, in which sunspots serve to coordinate the beliefs of agents on one vector of certainty equilibrium commodity prices.[1] A currently held belief is that even if a multiplicity of certainty equilibria is not necessary, it is sufficient for sunspot effects. The overall aim of this paper is to clear up these fallacies and to allow the theory to reveal that the origin of sunspot effects has no connection with a multiplicity of certainty equilibria.

This paper addresses the relation between sunspot effects and multiplicity within the class of incomplete markets models with numeraire assets and initial period consumption. The seminal Cass and Shell (1983) paper considered a form of restricted participation, whereas this paper focuses exclusively on incomplete markets. The model allows for initial period consumption for generality, and the results go far beyond generalizations of known results from models without initial period consumption (Mas-Colell, 1992; Hens, 2000). For a thorough accounting of the theory, I conduct analogous analyses for models without initial period consumption and provide the results in the footnotes of this paper. This will convince the reader that the results are not specific to the presence of initial period consumption.

The stochastic setting consists of two time periods with a finite number of states of uncertainty in the final period. The financial

☆ This work was previously circulated under the title "Sunspots and Multiplicity". I thank anonymous referees, Masanori Kashiwagi, Piero Gottardi, David Levine, Laurie Anderson, Alyson Price, seminar participants at UPF, and participants at the Cowles Yale and Max Weber Lustrum conferences for comments that greatly improved the paper. I acknowledge the support received from a Max Weber Fellowship (EUI). Comments are welcomed at mhoelle@purdue.edu.

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[1] This misunderstanding can perhaps be traced to Cass and Shell (1983). Most readers have tended to focus on the canonical example in the body of the paper, in which a multiplicity of certainty equilibria appears to be necessary for sunspot effects, rather than on the example in the Appendix proving a multiplicity is not necessary.

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http://dx.doi.org/10.1016/j.jmateco.2013.10.011

assets have payouts specified in the numeraire commodity (so-called numeraire assets), and these payouts are identical across the realizations of extrinsic uncertainty in the final period. The model does not contain intrinsic uncertainty, implying (i) that the only states of uncertainty are the states of extrinsic uncertainty and (ii) that the only independent numeraire asset is a risk-free bond with equal payouts in all states in the final period. These payouts are normalized to one.

For this model, I show that a multiplicity of certainty equilibria is neither necessary nor sufficient for sunspot effects. The fact that multiple certainty equilibria are not necessary for sunspot effects has been demonstrated in models other than the incomplete markets model considered in this paper.[2] Showing that a condition is neither necessary nor sufficient for a certain property requires only two well-chosen economies to serve as counter-examples. What I ultimately demonstrate in this paper is a general result, namely that both implications are false over a subset of economies commonly considered in the literature.

Numeraire assets, as considered in this paper, have different implications for sunspot equilibria than the two other asset types: nominal assets and real assets. With nominal assets, sunspot equilibria are generically indeterminate when there are fewer assets than states of extrinsic uncertainty (Cass, 1992). The intuition for this result is straightforward. In economies with market incompleteness equilibria are generically indeterminate, yet the set of equilibria without sunspot effects (namely those with identical real variables for all realizations of extrinsic uncertainty) is generically finite.

With real assets, over a generic subset of economies, sunspot effects occur without a multiplicity of certainty equilibria (Gottardi and Kajii, 1999). In their setup, the asset yields are identical across realizations of extrinsic uncertainty, but the asset payouts need not be. This is because the asset payouts, by definition, are functions of the endogenous commodity prices. In the terminology of Gottardi and Kajii (1999), the result states that a "potential multiplicity" of certainty equilibria is necessary for sunspot effects, a concept that I consider more formally in short order.

With numeraire assets, the sunspot equilibria are generically determinate (in contrast to nominal assets) and the asset payouts are parameters that are independent of endogenous commodity prices (in contrast to real assets).

For incomplete markets models with numeraire assets, is it possible to have sunspot effects even though a unique spot market equilibrium exists for all distributions of ex-post final period endowments? For clarity, ex-post final period endowments are the sum of the parameterized commodity endowments and the endogenous portfolio payouts. The answer to the previous question is yes, but only if the asset payouts depend upon the realizations of extrinsic uncertainty, as in Hens (2000).[3] This asset payout requirement violates the spirit of extrinsic uncertainty.

The following arguments are strongly made in Gottardi and Kajii (1999), but I summarize them for completeness. Provided asset yields are independent across states of extrinsic uncertainty, a necessary condition for sunspot effects is that multiple spot market equilibria exist for a nonempty subset of ex-post final period endowments. I call this multiplicity of spot market equilibria "ex-post multiplicity", which is an identical concept to the Gottardi and Kajii (1999) concept of "potential multiplicity". In models with and without initial period consumption, an equilibrium includes all household decisions, including those ex-ante decisions made before the uncertainty is revealed, specifically portfolio choices. The multiplicity of the entire certainty equilibrium vector is called "ex-ante multiplicity", or simply "multiplicity".

My first contribution is to complete the triangle begun by the two previous research contributions (Gottardi and Kajii, 1999; Hens, 2000) by providing an example with the concurrence of a unique certainty equilibrium and an equilibrium with sunspot effects. This example contains two elements: (i) numeraire assets whose payouts are equal for all realizations of extrinsic uncertainty and (ii) multiple spot market equilibria for some distribution of ex-post final period endowments. The example verifies that a multiplicity of certainty equilibria is not necessary for sunspot effects.

My second contribution is to provide an example with multiple certainty equilibrium and for which sunspot effects are impossible. This verifies that a multiplicity of certainty equilibria is not sufficient for sunspot effects.

For my third and main contribution, I analyze more deeply the relation between multiplicity and sunspot effects. I consider a subset of economies that are commonly analyzed in the sunspot literature. Within this subset, there exist economies with both a multiplicity of certainty equilibria and sunspot effects. Even so, the relationship between these two outcomes is only spurious, as there does not exist any theoretical link. The theoretical link is formally defined as a sunspot equilibrium being a "randomization" over the multiple certainty equilibria. By randomization, I only require that the sunspot equilibrium commodity prices in each state of extrinsic uncertainty correspond to the commodity prices from one of the certainty equilibria.[4] In summary, the result verifies that a multiplicity of certainty equilibria and sunspot effects are independent and unrelated theoretical concepts.

The paper is organized as follows. Section 2 introduces the model both with and without extrinsic uncertainty. Section 3 provides an example to show that a multiplicity of certainty equilibria is not necessary for sunspot effects. Section 4 provides an example to show that multiplicity is not sufficient. Section 5 provides a general result stating that an equilibrium with sunspot effects is never a randomization over certainty equilibria. Section 6 offers concluding remarks and the Appendix contains the proofs.

2. The model

2.1. Sunspot equilibrium

I consider a general equilibrium model with two time periods and extrinsic uncertainty in the final period. The extrinsic uncertainty is modeled as a finite number of states $s \in \mathbf{S} = \{1, \ldots, S\}$ that can be realized in the final period. By convention, the initial period is state $s = 0$. In all states, a finite number of households $h \in \mathbf{H} = \{1, \ldots, H\}$ trade and consume a finite number of physical commodities $l \in \mathbf{L} = \{1, \ldots, L\}$. The model is a financial model, as assets are present to allow households to transfer wealth between states.

[2] See Cass and Shell (1983) for two-period models of restricted participation, Azariadis and Guesnerie (1986) for OLG models of production, Peck and Shell (1991) for markets with information asymmetries, and Garratt et al. (2004) for nonconvex exchange economies.

[3] The initial attempt along these lines was made in Hens (2000), but an error was present in that paper's example. The error was pointed out by Barnett and Fisher (2002). Ultimately, an example is provided with more than two households, in which sunspot effects exist even though a unique spot market equilibrium exists for all distributions of ex-post final period endowments (see Hens et al. (2005), Proposition 2, which builds off of an insight in Hens and Pilgrim (2004)). In all these papers, the models do not allow for initial period consumption.

[4] Randomization, both the colloquial term and the technical definition given in this paper, is the intuition for sunspot effects from both: (i) Cass and Shell (1983), Observation 1 ("a sunspot equilibrium is constructed as a lottery over certainty equilibria", pg. 213) and (ii) Mas-Colell (1992) ("sunspots can matter only if they induce randomness", pg. 469).