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Prices as factors: Approximate aggregation with incomplete markets

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Abstract

Recent developments in intertemporal asset pricing theory focus on two sets of fundamental determinants of asset returns. Models with complete markets emphasize aggregate variables such as per capita consumption. Such models have not performed well empirically. Models with incomplete markets emphasize disaggregate variables such as the distribution of wealth. These models have shown empirical promise, but their lack of parsimony has led many to question their usefulness. Indeed, most empirical applications, as well as the *best practice* in the financial industry, ignore much of what theory has to say altogether. Practitioners favor factor models and, in particular, models in which asset returns serve as factors that explain other asset returns. This paper attempts to rationalize these very different approaches. We show that in an incomplete-markets economy—an economy in which the true pricing kernel is high dimensional and difficult to measure—a low-dimensional pricing kernel that depends on returns, as in the CAPM or the APT, can accurately price assets. In this sense, theory based on fundamentals admits an *approximate aggregation* that is consistent with practical application. Theory and practice, therefore, may not be as disparate as they might seem. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

A central goal of financial economics is to find a parsimonious representation of a *pricing kernel*, a positive random variable, M_t, that satisfies the no-arbitrage condition: $E_t(M_{t+1}R_{t+1})=1$, where R_{t+1} represents a vector of asset returns. Knowledge of the form of such a random variable is sufficient for the valuation of any state-contingent cash flow. A popular approach has been to put sufficient structure on a model economy so that M_t is a simple function of aggregate consumption, independent of the distribution of wealth. Lucas (1978), for example, studies a model in which M_t is coincident with the intertemporal marginal rate of substitution of a single, or representative, agent who consumes aggregate consumption in equilibrium. Constantinides (1982) shows that this representative-agent pricing kernel can result from a complete-markets equilibrium in which many different agents seek to maximize additively time-separable expected utility. Breeden (1979) and Grossman and Shiller (1982) show that in a continuous-time economy, or a discrete economy in which utility is quadratic, several other forms of heterogeneity are consistent with an aggregate consumption-based pricing kernel.

The quantitative shortcomings of specific parameterizations of the aggregate consumption version of M_t has led a number of authors (e.g., Aiyagari and Gertler, 1991; Alvarez and Jermann, 1999; Constantinides and Duffie, 1996; den Haan, 1994; Heaton and Lucas, 1996; Levine and Zame, 2000; Lucas, 1994; Mankiw, 1986; Marcet and Singleton, 1999; Telmer, 1993; Krusell and Smith, 1997; Zhang, 1997) to consider environments in which aggregation (in the sense of Constantinides, 1982) does not hold. In these environments frictions in the intertemporal trading mechanism—an insufficient number of state contingent claims, for instance—force agents to bear a portion of their idiosyncratic risk in equilibrium, thereby increasing the variability of M_t. Hansen and Jagannathan (1991) have shown that achieving a certain amount of variability in the pricing kernel is a necessary condition for understanding the behavior of excess returns on risky assets.

These asset-pricing theories have a number of potential benefits, the most obvious being that they provide formal models of the relationship between prices and quantities, thereby linking what we mean by risk to fundamentals such as preferences and technologies. Just as obvious, however, are the drawbacks associated with this approach. In many instances valuation can be high dimensional: a large number of state variables, the distribution of wealth for instance, are necessary for a characterization of equilibrium prices. This lack of parsimony stands in sharp contrast to a number of alternative valuation models that attain low dimensionality by assuming frictionless environments. Classic examples are the CAPM of Sharpe (1964) and Lintner (1965) and Ross's (1976) APT. More recent examples are factor-based approaches such as (among many others) Bossaerts and Green (1989), Chen et al. (1986),