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Arbitrage and price revelation with asymmetric information and incomplete markets

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Abstract

This paper deals with the issue of arbitrage with differential information and incomplete financial markets, with a focus on information that no-arbitrage asset prices can reveal. Time and uncertainty are represented by two periods and a finite set S of states of nature, one of which will prevail at the second period. Agents may operate limited financial transfers across periods and states via finitely many nominal assets. Each agent i has a private information about which state will prevail at the second period; this information is represented by a subset S_i of S. Agents receive no wrong information in the sense that the "true state" belongs to the "pooled information" set $\cap_i S_i$, hence assumed to be non-empty.

Our analysis is two-fold. We first extend the classical symmetric information analysis to the asymmetric setting, via a concept of no-arbitrage price. Second, we study how such no-arbitrage prices convey information to agents in a decentralized way. The main difference between the symmetric and the asymmetric settings stems from the fact that a classical no-arbitrage asset price (common to every agent) always exists in the first case, but no longer in the asymmetric one, thus allowing arbitrage opportunities. This is the main reason why agents may need to refine their information up to an information structure which precludes arbitrage.

Keywords: Asymmetric information; Private information; No-arbitrage; Incomplete markets; Refinement; Information revealed by prices

1. Introduction

In economies subject to uncertainty and asymmetric information, agents seek to infer relevant information from market indicators, such as prices, to refine their strategies. This

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issue is traditionally tackled by the so-called "rational expectations" models by assuming, quoting Radner (1979), that "agents have a "model" or "expectations" of how equilibrium prices are determined."

In this paper, agents learn from asset prices about partners' private information, by analyzing the arbitrage opportunities of the financial markets. They need not know the ex ante characteristics of the economy (preferences, endowments of other agents) or a defined relationship between prices and the collection of private information signals in the economy, as in the rational expectations' models. Thus, they are only required to know the market prices and their own characteristics. We define a notion of equilibrium, embedding the way agents infer information from asset prices, and its properties will be presented in a companion paper. For this purpose, however, we need first to study arbitrage theory with asymmetric information, which is the main aim of the present paper.

We consider the simplest tractable setting for the study of arbitrage. Time and uncertainty are represented by two periods ($t = 0$ and $t = 1$) and a finite set S of states of nature, one of which will prevail at the second period. Agents may operate limited financial transfers across periods and states via finitely many nominal assets. Each consumer receives a private information signal about which state will prevail at the second period. Asymmetric information is thus represented, for each agent i, by a subset S_i of S. Agents receive no wrong information in the sense that the "true state" belongs to the "pooled information" set $\cap_i S_i$, hence assumed to be non-empty. Similarly, when agents refine their information, i.e. when they infer a smaller set $\Sigma_i \subset S_i$, they also receive no wrong signal, so that $\cap_i \Sigma_i \neq \emptyset$. This is guaranteed, in particular, when the refinement (Σ_i) of the collection (S_i) preserves its pooled information, that is, $\cap_i S_i = \cap_i \Sigma_i$.

Our analysis is two-fold. We first extend the classical non-arbitrage analysis to the asymmetric setting, via a concept of no-arbitrage price, and second, we study how such no-arbitrage prices convey information to agents in a decentralized way. The main difference between the symmetric and asymmetric settings stems from the fact that a classical no-arbitrage asset price (common to every agent) always exists in the symmetric case, but no longer in the asymmetric one, thus allowing arbitrage opportunities. This is the main reason why agents may need to refine their information up to an information structure precluding arbitrage.

The paper is organized as follows. In Section 2, we present the framework and recall the basic concepts of arbitrage-free information structures, refinements, and no-arbitrage prices. We also define the notion of financial equilibrium in an asymmetric setting, which explicitly presents consumers' behavior and the need for a refinement of information when it is not arbitrage-free at the outset (Definition 2.2). In Section 3, we characterize arbitrage-free structures by the absence of future (i.e. at $t = 1$) arbitrage opportunities on the financial market, called the AFAO property (Proposition 3.1). We show that every information structure (S_i) has a unique coarsest arbitrage-free refinement, denoted by $(\bar{\boldsymbol{S}}_i)$, which does not contain any wrong signals since $\cap_i S_i = \cap_i \bar{\boldsymbol{S}}_i$ (Proposition 3.2). We end the section with the relationship between "fully-revealing" structures, i.e. such that $\bar{\boldsymbol{S}}_i$ coincides with agents' pooled information, market completeness (Proposition 3.3) and symmetric information (Proposition 3.4). In Section 4, we first define, for every agent i and every asset price q, the "revealed information set" $\boldsymbol{S}_i(q) \subset S_i$ (Definition 4.1). We then define an extended notion of no-arbitrage asset price (Definition 4.2), as the common no-arbitrage