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Pricing and hedging in incomplete markets ☆

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Abstract

We present a new approach for positioning, pricing, and hedging in incomplete markets that bridges standard arbitrage pricing and expected utility maximization. Our approach for determining whether an investor should undertake a particular position involves specifying a set of probability measures and associated floors which expected payoffs must exceed in order for the investor to consider the hedged and financed investment to be acceptable. By assuming that the liquid assets are priced so that each portfolio of assets has negative expected return under at least one measure, we derive a counterpart to the first fundamental theorem of asset pricing. We also derive a counterpart to the second fundamental theorem, which leads to unique derivative security pricing and hedging even though markets are incomplete. For products that are not spanned by the liquid assets of the economy, we show how our methodology provides more realistic bid–ask spreads. © 2001 Published by Elsevier Science S.A.

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1. Introduction

Opportunities to transform initial certain wealth into random future wealth abound in a thriving economy. Thus, a fundamental problem in investment theory concerns the issue of whether or not to undertake such an opportunity. To simplify the problem, one often assumes that the opportunity vanishes forever if it is initially rejected. For further simplification, the scale of the opportunity is usually taken to be fixed, at least at the outset.

The purpose of this paper is to propose a new approach for deciding whether or not to accept opportunities of this type. Our approach is intermediate between expected utility theory and arbitrage pricing theory in terms of both the initial information required and the power of the conclusions derived. Hence, to place our approach in the proper context, we now review these two fundamental paradigms.

Expected utility maximization is a powerful tool for deciding whether or not to accept a project at a given time and scale. So long as an investor's behavior is consistent with the von Neumann Morgenstern axioms (see Varian, 1984), an investor accepts an opportunity if and only if it increases her expected utility. While expected utility maximization has a long history and a strong theoretical appeal, it has had limited acceptance in practice. While this negative result could be due to real-world violation of the behavioral axioms, a more compelling reason involves the difficulty inherent in specifying the required inputs to the optimization. These inputs include the current endowment, the joint stochastic process over all assets, and the utility function over all certain wealth levels. In practice, corporations typically fail to fully specify these three fundamental constructs in making capital budgeting decisions, and our experience is that even professional investors are generally unwilling to explicitly specify these three inputs when making investment decisions. Even if the constructs are inferred through past decisions, the revealed constructs are often inconsistent over time and across assets.[1] This inconsistency would be benign if the recommended action were robust to the particular specification of endowments, beliefs, and preferences. Unfortunately, the maximization is notoriously sensitive to these inputs, whose formulation is suspect at the outset. This shortcoming renders the methodology potentially useless, primarily because the decisions consistent with the inputs used in the optimization may be seriously disputed by other perspectives.

To draw some inferences on these other perspectives, it is widely acknowledged that market prices of related instruments provide useful informational inputs for the decision process. So long as these prices are liquid, their levels reflect a panoply of endowments, beliefs, and preferences. If an opportunity is undertaken, the relevance of related liquid market prices is enhanced by the

[1] See the substantial literature on the Allais (1987) paradox.