Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
Journal of Mathematical Economics 41 (2005) 43–66
JOURNAL OF Mathematical ECONOMICS
www.elsevier.com/locate/jmateco

Evolutionary stability of portfolio rules in incomplete markets

Thorsten Hens [a,*], Klaus Reiner Schenk-Hoppé [a,b]

[a] *Institute for Empirical Research in Economics, University of Zurich, Blümlisalpstrasse 10, CH-8006 Zürich, Switzerland*
[b] *Institute of Economics, University of Copenhagen, Studiestræde 6, DK-1455 Copenhagen K, Denmark*

Received 22 October 2002; accepted 21 January 2003
Available online 2 July 2004

Abstract

This paper studies the evolution of wealth shares of portfolio rules in incomplete markets with short-lived assets. Prices are determined endogenously. The performance of a portfolio rule in the process of repeated reinvestment of wealth is determined by the wealth share eventually conquered in competition with other portfolio rules. Using random dynamical systems theory, we derive necessary and sufficient conditions for the evolutionary stability of portfolio rules. In the case of Markov (in particular i.i.d.) payoffs these local stability conditions lead to a simple portfolio rule that is the unique evolutionary stable strategy. This rule possesses an explicit representation. Moreover, it is demonstrated that mean–variance optimization is not evolutionary stable while the CAPM-rule always imitates the best portfolio rule and survives.

JEL classification: D52; D81; D83; G11

Keywords: Evolutionary finance; Portfolio theory; Market selection; Incomplete markets

1. Introduction

We consider an incomplete asset market where a finite number of portfolio rules manage capital by iteratively reinvesting in a fixed set of assets. Assets are short lived but identically "re-born" in every period. Their payoffs depend on a stationary process in discrete time. Portfolio rules are encoded as non-negative vectors of expenditure shares for assets which

* Corresponding author. Tel.: +41 1634 3706; fax: +41 1634 4907.
E-mail addresses: thens@iew.unizh.ch (T. Hens), klaus@econ.ku.dk (K.R. Schenk-Hoppé).

0304-4068/$ – see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.jmateco.2003.01.001

may depend on past observations. The set of portfolio rules we consider is not restricted to those generated by expected utility maximization. It may as well include investment rules favored by behavioral finance models. Indeed any portfolio rule that is adapted to the information filtration is allowed in our framework. In every period in time the available market capital is given by the total payoff of the assets. Portfolio rules compete for this market capital—the endogenous price process provides a *market selection mechanism* along which some strategies gain market capital while others lose. The particular case of Arrow securities and simple, i.e. constant over time, portfolio rules has been introduced in the seminal paper of Blume and Easley (1992).

The framework studied here can be seen as a Darwinian theory of asset markets.[1] This theory views asset markets as being stratified according to the portfolio rules that investors use to manage wealth. With every such rule (mean–variance rule, growth-optimal rule, CAPM-rule, naive diversification, prospect theory based rules, relative-dividends rule, for example) a certain amount of wealth is being managed. In our model the impact of any such rule on market prices is proportional to the amount of wealth managed by the rule. In a Darwinian model two forces are at work, one which reduces the variety of species and one which increases it. In our model the first such force is the endogenous return process which is a market selection mechanism that determines the evolution of wealth managed by the portfolio rules. Secondly, any system of portfolio rules that is selected by the market selection process is checked for its evolutionary stability, i.e. it is checked whether the innovation of a new portfolio rule with initially very little wealth can grow against the incumbent rules. The Darwinian theory of asset markets seems to describe very well a modern asset market in which most of the available capital is invested by delegated management. Indeed investors typically choose funds by the portfolio rules, or styles, according to which the money is invested. Style consistency is nowadays one of the most important features in monitoring fund managers.

In this paper we derive a description of the market selection process from a random dynamical systems perspective. In each period in time, the evolution of the distribution of market capital (wealth shares) is determined by a map that depends on the exogenous process determining the asset payoffs. An equilibrium in this model is provided by a distribution of wealth shares across portfolio rules that is invariant under the market selection process. It turns out that (provided there are no redundant assets) every invariant distribution of market shares is generated by a *monomorphic* population, i.e. all traders with strictly positive wealth use the same portfolio rule. A criterion for evolutionary stability as well as evolutionary instability is derived for such monomorphic populations. Roughly speaking a portfolio rule is evolutionary stable if it has the highest exponential growth rate in any population where itself determines market prices. This implies that an evolutionary stable investment strategy is robustness against the entry of new portfo-

[1] Veblen (1898) asked more than 100 years ago “Why is economics not an evolutionary science?,” and Alfred Marshall states in the preface of his *Principles of Economics* (8th edition), “The Mecca of the economist lies in economic biology rather than in economic dynamics.” (It is noteworthy to point out that the term “economic dynamics” is meant in a different sense than it is used today.) However, application of evolutionary reasoning to financial markets is quite recent, see e.g. Farmer and Lo (1999).