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An extension of Mantel (1976) to incomplete markets

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Abstract

In the incomplete markets model with numeraire asset and a single consumption good we show that even with homothetic preferences, on compact sets of prices continuity, Walras' identity and homogeneity characterize the properties of market excess demand. This result is proved by an extension of [J. Econ. Theory 12 (1976) 197] to the case of incomplete markets. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

In this note, we show that under some conditions, even when markets are incomplete, on compact sets of prices continuity, Walras' law and homogeneity characterize the properties of market excess demand.

For the complete markets model this statement has been shown to be true in a series of papers starting from Sonnenschein (1973a,b), Debreu (1974) and Mantel (1974, 1976). For the incomplete markets model, recently similar results have been found. Bottazzi and Hens (1996), Gottardi and Hens (1999), Gottardi and Mas-Colell (2000) and Chiappori and Ekeland (1999) demonstrate that by restricting attention to a point or to small open neighborhoods of regular prices these results carry over to the case of real assets, the case of nominal assets and to the case of demand being defined on the Grassmanian, respectively.

The result presented in this note holds for arbitrary compact sets of prices, however, we restrict attention to the incomplete markets model with real assets and a single consumption

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good in each state. This case is of particular importance for financial economics. Whereas Debreu's (1974) proof cannot be applied to the incomplete markets setting in order to obtain a global decomposition result, as we demonstrate in Appendix A, we show that Mantel's (1976) proof based on duality theory nicely carries over to the setting of this note. Indeed, following Mantel's (1976) approach one can generate any excess demand even with homothetic preferences. The case of homothetic preferences is of particular importance for aggregation results because in this case demand has a lot of structure. If utility functions were identical or if endowments were co-linear then, in the case of homothetic preferences, market demand is not arbitrary but can be generated by a representative consumer. For the incomplete markets model aggregation in the case of homothetic and identical preferences has been shown by Hens (1990) and by Detemple and Gottardi (1998), generalizing the theorems of Antonelli (1886) and of Gorman (1953), respectively. The case of homothetic preferences and co-linear endowments has been proved for the incomplete markets model by Voß (1997), who generalizes a result of Chipman (1974).

2. Mantel's theorem

In order to put our result into the correct perspective, we will briefly recall Mantel's theorem.

Let $\mathbb{R}^n$ be the commodity space and let $H := \left\{ p \in \mathbb{R}^n \,\middle|\, \sum_{l=1}^n p_l = 1 \right\}$ be the set of normalized prices. For every compact convex subset $P \subset H$ Mantel defines $P^* := \{x \in \mathbb{R}^n | px \geq 0 \text{ for all } p \in P\}$ the positive polar of the set P. Given these definitions we can state

Theorem (Mantel (1976)). *Let $P \subset H$ be compact, convex. Let $Z : P \to \mathbb{R}^n$ be C^2 on P and satisfy $pZ(p) = 0$ for all $p \in P$. Let $\omega^i \in P^*$, $i = 1, \ldots, n$ be independent vectors. Then there exists a real $k > 0$, a convex cone $X \subset P^*$, and n unsatiated consumers with strictly concave, homogeneous utility functions$U^i : X \to \mathbb{R}$ and initial endowments $k\omega^i$, whose excess demand functions add up to Z on P.*

If one wants to apply Mantel's theorem to the standard Arrow–Debreu model (see e.g. Theorem 7 in Shafer and Sonnenschein (1982)) one could set $P = \{p \in \mathbb{R}^n | p_l \geq \epsilon, l = 1, \ldots, n \sum_{l=1}^n p_l = 1\}$ and gets that P^* comes close to the nonnegative orthant for small $\epsilon > 0$.

3. The model

In the following, we apply Mantel's theorem to the market excess demand for assets in an incomplete markets model with a single consumption good, numeraire assets and two periods. To this end let there be a single consumption good available in the second period. There is symmetric uncertainty because in the second period one of $s = 1, \ldots, S$ states realizes. Consumers $i = 1, \ldots, I$ are endowed with state contingent resources $\omega^i \in \mathbb{R}^S_+$ and they evaluate state contingent consumption x^i according to continuous, monotone and strictly quasi-concave utility functions $U^i : \mathbb{R}^S_{++} \mapsto \mathbb{R}$. To insure against the uncertainty, in