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Evaluation of insurance products with guarantee in incomplete markets

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Abstract

Life insurance products are usually equipped with minimum guarantee and bonus provision options. The pricing of such claims is of vital importance for the insurance industry. Risk management, strategic asset allocation, and product design depend on the correct evaluation of the written options. Also regulators are interested in such issues since they have to be aware of the possible scenarios that the overall industry will face. Pricing techniques based on the Black & Scholes paradigm are often used, however, the hypotheses underneath this model are rarely met.

To overcome Black & Scholes limitations, we develop a stochastic programming model to determine the fair price of the minimum guarantee and bonus provision options. We show that such a model covers the most relevant sources of incompleteness accounted in the financial and insurance literature. We provide extensive empirical analyses to highlight the effect of incompleteness on the fair value of the option, and show how the whole framework can be used as a valuable normative tool for insurance companies and regulators.

Keywords: Life insurance; Policies with minimum guarantee; Option pricing; Incomplete markets

1. Introduction

In recent years, embedded options in life insurance contracts became subject to increasing concern for the academic world as well as for practitioners. The consequences of failing to properly price the embedded options in insurance contracts became evident after the case "Equitable Life vs Hyman", where the insurance company had to close its funds after suffering substantial losses due to a decision of the House of Lords interpreting negatively the discretion with which Equitable had structured the bonus to the policyholders. In order to avoid such occurrences, the new International Financial Reporting Standards for insurance contracts (IFRS 4) and Solvency II now require insurance companies to measure and price embedded derivatives in insurance contracts at a fair value.

In this paper we focus on the evaluation of life insurance products with embedded options originated by minimum guarantee returns and bonus provision. The option pricing approach has been widely used to determine the fair price of a large range of products marketed by life insurance companies and pension funds (see Babbel and Merril (1998), Boyle and Hardy (1997), Brennan and Schwartz (1979), Embrechts (2000), Vanderhoof and Altman (1998)).

The advance in this field has yielded numerous studies whose primary goal is to properly evaluate complex bonus mechanism, introducing surrender options (turning the option to an American-type), and refining the stochastic framework (see Bacinello (2003), Giraldi et al. (2003), Grosen and Jørgensen (2000, 2002), Miltersen and Persson (1999)).

All these authors develop their models within the framework outlined by the main assumptions of the option pricing theory, i.e., no-arbitrage, dynamic hedging, and market completeness. Of these three hypotheses, the least realistic one is that of market completeness, namely, it is possible to replicate the payoff of any claim in the market by means of a self-financing strategy.

There are manifold sources of market incompleteness. For example:

1. Jumps in the underlying stochastic process due to bubbles-economy crash, nature/weather-catastrophic large claim;
2. Heteroscedasticity of the processes for the underlying assets;
3. Market frictions: short sales, transaction costs, operational constraints;

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4. Non-tradeability of the underlying asset due to the absence or lack of liquidity of the reference market. This is especially true when the reference fund is an internal portfolio of the insurance company;
5. Discrete hedging, given that continuous rebalancing is unrealistic and expensive;
6. Mortality risk, that is, the risk associated with not knowing how many of the policyholders will survive.

With regard to the insurance field, only few authors concentrate their studies on the issue of market incompleteness. Møller (2001) determines risk-minimizing hedging strategies for equity-linked pure endowment contracts. In this case, the incompleteness arises from mortality risk, that is an additional risk factor, independent of the financial market risk. The financial market itself is assumed to be complete, and the guaranteed option written in the insurance contract is hedged as in the Black & Scholes model. Further extensions can be found in Møller (2002), where the author compares results obtained with super-replication (El Karoui and Quenez, 1995), mean–variance hedging (Duffie and Richardson, 1991), risk minimization (Föllmer and Sondermann, 1986), and indifference pricing. The latter approach is related to the indifference price of the contract under different filtrations, which are associated to different information sets (see Møller, 2003).

Moore and Young (2003) employ a utility method to determine the price of endowment contracts linked to risky index. In this case too, the source of incompleteness is the mortality risk. Under the principle of equivalent utility, the premium is that price which leaves the insurer indifferent between writing and not writing the endowment contract. They prove that, under the assumption of exponential utility, the indifference premium solves a nonlinear partial differential equation, where the nonlinear term reflects the additional mortality risk and the exponential risk preferences of the model.

Coleman et al. (2006) cope with the same problem and solve it in a more general setting by addressing market incompleteness in the many facets summarized above. They model the dynamics of the objective price measure by merging the traditional Black & Scholes price process with the Merton's jump diffusion process. They then hedge the insurance claim using the underlying asset and a set of standard options expiring before the maturity of the claim. The hedging strategy is determined by applying the minimum local hedging risk principle by Föllmer and Schweizer (1989). Through a Montecarlo simulation, they show that the risk-minimization hedging strategy delivers better performances with respect to the Black & Scholes delta hedging.

The main contribution of our paper is to extend the analysis developed in Briys and de Varenne (2001), and nailed down in Grosen and Jørgensen (2002), to encompass the sources of market incompleteness listed above. We assume that the equity holders of the insurance company have limited liability, and thus we properly model the issue of insolvency risk due to the bankruptcy event.

We use a stochastic programming model (King, 2002) to super-replicate the payoff generated by the bonus distribution scheme. As we will show, the model is general enough to deal with any complex final payoff generated by European path-dependent options. We account for the bankruptcy event by considering the liabilities of the company as a risky (defaultable) bond. Following Grosen and Jørgensen (2002), we introduce regulatory restrictions assuming that the solvency of the company is monitored at discrete points in time.

The paper is organized as follows: Section 2 defines the basic framework and the specifications of the insurance contract. Section 3 shows how stochastic programming models can handle option pricing in incomplete markets, and provides a framework to hedge the payoff generated by the insurance bonus scheme. Section 4 describes the experimental setting used to implement the model and discusses the results obtained. The final section contains our conclusions as well as some suggestions for future research.

2. Insurance products with guarantee

We assume that an insurance company issues contracts that promise to pay some benefits, at the end of a specified maturity time T, contingent to the value of a reference fund I_T. More general payout schedules can be introduced without changing the main body of the model and its tractability.

We denote by I_0 the value of the fund at the inception of the contract, and we let $L_0 \equiv \alpha I_0$ be the premium paid by the policyholders to enter the contract; the initial investment by the equityholders is then given by $E_0 \equiv (1-\alpha) I_0$.

Note that, unlike Briys and de Varenne (2001) and Grosen and Jørgensen (2002), the reference fund could be any index used to determine the contractual obligations of the company. Broadly speaking, what matters for the company are the liabilities generated by the final payoff, and its main concern is the hedging of such a claim.

As stated above, a major source of incompleteness is the non-tradeability of the underlying asset or liquidity restrictions on it. For this reason, the hedging portfolio will, in general, consist of liquid assets (stock, bonds, options or futures) other than the underlying asset. In case of illiquidity or non-tradeability of the underlying, this hypothesis is more realistic than assuming that the hedging is performed by trading the underlying and the risk-free.

The insurance contract is equipped with a minimum guarantee provision. In particular, at maturity, the policyholder will receive an amount of money, L_T^G, obtained by compounding the initial premium, L_0, at the rate r_G,

$$L_T^G = L_0 \, \mathrm{e}^{r_G T}. \tag{1}$$

Besides the final maturity guarantee, a bonus provision entitles policyholders to receive a share of the upside potential over the guarantee payment. The payoff of the bonus option is given by

$$\delta \left[\alpha I_T - L_T^G \right]^+, \tag{2}$$

where δ is the participation coefficient and $[\cdot]^+$ indicates the positive part of its content.