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Comparison of solutions to the incomplete markets model with aggregate uncertainty

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ABSTRACT

This paper compares numerical solutions to the model of Krusell and Smith [1998. Income and wealth heterogeneity in the macroeconomy. Journal of Political Economy 106, 867–896] generated by different algorithms. The algorithms have very similar implications for the correlations between different variables. Larger differences are observed for (i) the unconditional means and standard deviations of individual variables, (ii) the behavior of individual agents during particularly bad times, (iii) the volatility of the per capita capital stock, and (iv) the behavior of the higher-order moments of the cross-sectional distribution. For example, the two algorithms that differ the most from each other generate individual consumption series that have an average (maximum) difference of 1.63% (11.4%).

1. Introduction

This paper compares different algorithms to solve the model of Krusell and Smith (1998), a popular model with a continuum of heterogeneous agents, idiosyncratic as well as aggregate risk, incomplete markets, and an inequality constraint on the chosen capital level.[1] Models with heterogeneous agents and incomplete markets are becoming more and more important in both macro and finance. Consequently, it is important that we know how to solve them well. This is not a trivial task, because (i) the set of state variables includes the cross-sectional distribution of income and wealth levels, a high dimensional object, (ii) the amount of idiosyncratic uncertainty is quite high so nonlinearities are likely to matter, and (iii) the occasionally binding constraint results in non-differentiable policy functions.

There are now several algorithms that can solve this type of model and in this paper I investigate whether the different algorithms generate similar answers when solving the model of Krusell and Smith (1998). The solutions turn out to differ substantially in several dimensions. This is especially true for the individual choices. Not only do the generated series differ during exceptional periods, such as particularly bad times, but there are even nontrivial differences between the implied first moments. Several accuracy checks are performed. Overall, the algorithm of Reiter (2009b) performs best in terms of accuracy. It clearly performs the best in terms of the accuracy of the individual policy rules and the accuracy of its aggregate law of motion is close to the most accurate aggregate laws of motion, which are the ones obtained with the Krusell–Smith algorithm. The performance of the algorithm of den Haan and Rendahl (2009) is close to the performance of Reiter (2009b) in terms of accuracy, but slightly worse. Interestingly, the algorithms of den Haan and Rendahl (2009) and

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[1] The actual version of the model considered and its parameter values can be found in den Haan, Judd and Juillard et al. (2009).

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Reiter (2009b) are also the fastest, with the algorithm of den Haan and Rendahl (2009) roughly seven times as fast as the algorithm of Reiter (2009b).[2]

The model considered here is a nontrivial model, but there are much more complex models considered in the literature. The fact that the different algorithms generate results that are not that similar, should motivate us to be careful in numerically solving these models. There are some useful lessons that can be learned from this comparison project. Those are the following:

- It is essential to have an algorithm for the individual problem that does well in terms of accuracy as well as speed. Standard lessons from the numerical literature, for example, that time iteration is typically faster and more reliable than fixed-point iteration, should not be ignored.[3] Also, the lower and upper bounds of the grid should be chosen with care in order not to exclude useful grid points. Grid points should also not be wasted; for this project, I find that the algorithms that use the largest range for individual capital also have lower accuracy. Finally, the method of endogenous grid points, proposed in Carroll (2006), is recommended. It is not clear whether this leads to a more accurate solution, but it is definitely faster and makes it, for example, easy to implement time iteration.
- It is important to realize that the properties of an algorithm found when solving for individual policy rules in the model *without* aggregate uncertainty, i.e., for a fixed capital stock level, do not carry over to the model *with* aggregate uncertainty, even when taking as given the law of motion for aggregate capital. In particular, this paper shows that it is more difficult to get accurate individual policy rules in the model with than in the model without aggregate uncertainty (even when taking the aggregate law of motion as given).
- In solving models with a representative agent, it is typically possible to achieve arbitrary accuracy. None of the algorithms considered here do extremely well in terms of all the accuracy tests. Especially the outcomes of the accuracy test for the aggregate policy rule are somewhat disappointing.[4] The maximum errors in a simulation of 10,000 observations vary across algorithms from 0.156% to 1.059%. Ideally, these should be at least a factor 10 smaller than the lowest values generated here.
- Given that it is not (yet) easy to generate numerical solutions with arbitrary accuracy, it is important to perform accuracy tests. The role of a good accuracy procedure consists not only of providing a measurement of the accuracy of the solution, but also of making clear which aspect of the solution is inaccurate when and whether the inaccuracies found matter.
- Algorithms and computers will get better and the model considered in this paper will hopefully soon be solved with arbitrary accuracy. But models are likely to get more complex at a faster rate, so numerical solutions will be generated that—like those considered in this paper—do reasonably well, but not exceptionally well in all accuracy tests considered. The question is what to do in such cases. Numerical solutions can fail accuracy tests and still give the right answer to the question the researcher is interested in.[5] The best accuracy test is, therefore, to "play around" with different choices, such as different classes of approximating functions and/or different grids, and to see whether the results of interest change. It would be even more convincing if the results do not change if a different algorithm is used to solve the model. Algorithms differ substantially in their programming burden. But both the popular Krusell–Smith algorithm and the recently developed algorithm of den Haan and Rendahl (2009) are quite simple to program, so the researcher does no longer have a good excuse not to try more than one algorithm.

This paper is organized as follows. In Section 2, I give a brief overview of the different algorithms used. In Section 3, I discuss the differences between the algorithms in solving a model without aggregate uncertainty and in Section 4 I discuss the differences between the algorithms in solving the full model, that is, the model with aggregate uncertainty. The last section concludes.

2. Different types of algorithms

Recursive numerical solutions of DSGE models consist of functions of the state variables. Existing algorithms are based on either the projection method or the perturbation method, sometimes on both. The projection method consists of two steps. In the first step, a grid of the state variables is constructed and one defines at each grid point error terms that provide a measure for the fit of any approximating function.[6] The second step consists of choosing the coefficients of the numerical approximation to obtain the best fit for a given loss function of the error terms.[7] The perturbation approach solves for the

[2] The algorithm of Kim et al. (2009) is even faster, but this algorithm does not solve the actual model specified.

[3] See Chapter 16 in Judd (1998) for a discussion.

[4] This paper uses a more demanding (but much better) accuracy test than the R^2 that is typically used in the literature.

[5] Some accuracy tests are directly linked to properties of interest. Santos (2000) relates the Euler equation residual to errors in the policy function. Reiter (2001) and Santos and Peralta-Alva (2005) construct a relationship between the size of the errors found and upperbounds on the errors of objects that economists are interested in such as the obtained utility level or moments.

[6] Numerical procedures, such as quadrature methods to calculate conditional expectations, may still be needed to calculate the value of the error terms.

[7] Or obtain a perfect fit if there are as many grid points as unknown coefficients.