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Journal of Mathematical Economics 42 (2006) 255–268
JOURNAL OF Mathematical ECONOMICS
www.elsevier.com/locate/jmateco

Existence of equilibrium and price adjustments in a finance economy with incomplete markets

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Received 26 March 2003; received in revised form 18 June 2004; accepted 22 February 2005
Available online 7 March 2006

Abstract

In this paper the standard two-period general equilibrium model with incomplete financial markets is considered. First, existence of equilibrium is proved using a stationary point argument on the set of no-arbitrage prices. Prices are normalised with respect to the market portfolio. The proof does not use the commonly applied normalization on the unit sphere or truncation of the set of prices. Also a new price adjustment process is proposed. The process generates a path of price vectors from an arbitrary price vector to an equilibrium. The path can be followed by a simplicial algorithm for finding stationary points on polyhedra.

Keywords: General equilibrium; Incomplete markets; Price adjustments

1. Introduction

The main focus of this paper is to describe a price-adjustment process in an economy with incomplete financial markets, that converges to an equilibrium price vector. It turns out that the simplicial algorithm for calculating stationary points of a continuous function on a polytope as developed by Talman and Yamamoto (1989) can be used to describe price formation on financial markets.

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0304-4068/$ – see front matter
doi:10.1016/j.jmateco.2005.02.002

In this paper the simplest general equilibrium model with incomplete markets as is presented in, e.g. Magill and Quinzii (1996) is considered. There are two periods of time (present and future), a finite number of possible future states, one consumption good and a number of financial securities that can be used to transfer income from the present to the future. For the consumption good there are spot markets, so at present one cannot trade the consumption good for the future. Financial markets are incomplete if not all possible income streams for present and future can be attained by trading in the assets available on the existing financial markets.

Existence of equilibrium in a two-period general equilibrium model with multiple consumption goods and (possibly) incomplete markets is proved in Geanakoplos and Polemarchakis (1986). They prove existence on the set of no-arbitrage prices. These are prices such that it is impossible to create a portfolio of assets which generates a non-negative income stream in the future and has non-positive costs at present. The proof uses a fixed point argument for functions on compact sets. For that, since the set of no-arbitrage prices can be unbounded, the proof of Geanakoplos and Polemarchakis (1986) uses a compact truncation of this set. In this paper we present an existence proof for the one consumption good model that uses a stationary point argument without truncating the set of no-arbitrage prices. Other existence proofs use some transformation of the underlying model. Hens (1991), for example, introduces an artificial asset to translate the present into the future. The approach taken by Hirsch et al. (1990) shows existence of equilibrium in a model with only state prices. Then it is argued that each equilibrium in the original model corresponds one-to-one to an equilibrium in state prices.

Given that an equilibrium exists the question arises how to compute one. There is a homotopy method introduced in Herings and Kubler (2002) that requires differentiability assumptions on the utility functions. In this paper we show that one can use the simplicial approach developed by Talman and Yamamoto (1989), which does not require additional assumptions to the ones needed to prove existence. The algorithm generates a piecewise linear path and approximately follows the piecewise smooth path of a price adjustment process. The latter process connects an arbitrarily chosen initial price vector with an equilibrium price vector. Note that the Talman and Yamamoto (1989) algorithm is defined for functions on polytopes. The set of no-arbitrage prices for the model can, however, be an unbounded polyhedron. Therefore, we extend the algorithm of Talman and Yamamoto (1989) to unbounded polyhedra. In the literature there are simplicial algorithms for functions on possibly unbounded polyhedra, notably by Dai et al. (1991) and Dai and Talman (1993). These algorithms cannot be applied, however, since they assume pointedness of the polyhedron and affine functions, respectively.

The paper is organised as follows. In Section 2 the economic model is described. In Section 3 we prove the existence of equilibrium and in Section 4 we adapt the simplicial algorithm of Talman and Yamamoto (1989) for the two-period finance economy. In Section 5 the algorithm is presented in some detail and illustrated by means of a numerical example.

2. The finance economy

The general equilibrium model with incomplete markets (GEI) explicitly includes incomplete financial markets in a general equilibrium framework. In this paper the simplest version is used. It consists of two time periods, $t = 0, 1$, where $t = 0$ denotes the present and $t = 1$ denotes the future. At $t = 0$ the state of nature is known to be $s = 0$. The state of nature at $t = 1$ is