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# Slack in incomplete markets with nominal assets: A symmetric proof

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**Abstract**

In this paper, we provide an equilibrium analysis in the framework of incomplete markets where some agents' preferences are possibly satiated at some state of the nature. We will consider nominal assets with exogenously fixed asset prices. We extend the notion of equilibrium with slack – introduced by Drèze and Müller [Drèze, J., Müller, H., 1980. Optimality properties of rationing schemes. Journal of Economic Theory 23, 150–159] in a fixed price setting – to the GEI framework.

If the preferences satisfy the non-satiation property at each state of the nature, then our existence result of equilibria with slack leads to the existence of an ordinary financial equilibrium. It is worth to stress that unlike Cass [Cass, D., 1984. Competitive Equilibrium with Incomplete Financial Markets, CARESS W 84-09; Cass, D., 2006. Journal of Mathematical Economics 42, 384–405], our existence proof does not break the symmetry of the problem.


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## 1. Introduction

In the general equilibrium model with incomplete markets, the existence of equilibria relies on the assumption that investors are non-satiated at each state of the nature. This means that it is always possible for each state of the nature to change consumption for this state, keeping it fixed for the other states, and increase thereby utility.

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In the capital asset pricing model (CAPM), the utility functions are in general assumed to depend only on the mean and the variance of the wealth associated to a portfolio. This is true if the utility is of the von Neumann–Morgenstern type with a quadratic utility function or if the revenue is normally distributed. One easily checks that this type of utility functions does not satisfy the non-satiation property at each state of the nature and thus an equilibrium may fail to exist.

In the complete market setting an equilibrium may fail to exist without the non-satiation property. At any given price some of the consumers may want to consume in the interior of his consumption set.[1] Thus, Walras' law and therefore equilibrium existence fails. Fixed price setting (Drèze and Müller, 1980) and the absence of the strong survival assumption may lead to the same problems, even if preferences are locally non-satiated.

The existence problem can be overcome by allowing some agents to spend more than the value of their initial endowments. Such an equilibrium is called dividend equilibrium or equilibrium with slack. It was first introduced in a fixed price setting in Drèze and Müller (1980) and this was later adapted to the standard model of complete markets by Makarov (1981); Aumann and Drèze (1986), and Mas-Colell (1992). As pointed out by Kahji (1996) the slack in consumers income may be interpreted as the value of paper money which is held by the consumers as initial endowment. The value of this paper money may then be positive if non-satiation fails to hold.

We will adapt the notion of equilibrium with slack to the case of incomplete markets. Some slack in the income will be allowed at each state of the nature.

As will be remarked later, the set of equilibria with slack and particularly the set of the corresponding allocations, is not modified if a useless commodity is introduced in the economy. This property is not true for the (ordinary) equilibria if the preferences are possibly satiated. Indeed, in the case of satiated preferences, an equilibrium may still exist, but as shown by the following example, allowing the introduction of a useless commodity, new equilibrium allocations may occur and may even improve the utility level of each consumer.

**Example 1.1.** There are two periods $t = 0$ and $t = 1$ and one physical commodity per period, no financial asset is available which would allow wealth transfer across dates. The two consumers are respectively endowed with $e_1 = (3, 1)$ and $e_2 = (1, 3)$. Their preferences exhibit strong complementarity between present and future consumption represented by the utility functions $u_1(x) = u_2(x) = \min\{x_1, x_2\}$. There are three spot-equilibria : $x_i = e_i$ for $i = 1, 2$ with $p \gg 0$; $x_1 = (3, 3)$, $x_2 = (1, 1)$ with $p = (1, 0)$ and $x_1 = (1, 1)$, $x_2 = (3, 3)$ with $p = (0, 1)$. Now introduce for each state a second commodity and allocate one unit to each consumer at each date. Thus, $\tilde{e}_1 = (3, 1, 1, 1)$, $\tilde{e}_2 = (1, 1, 3, 1)$. The preferences do not depend on the amount of the new good held by the agents. The set of spot-equilibria contains the previous equilibria with each agent keeping his endowment in the new commodity and the price of the new commodity set equal to zero. However the following is now also an equilibrium: $x_1 = (2, 2, 2, 0)$, $x_2 = (2, 0, 2, 2)$ with $p = (1, 1, 1, 1)$. The new equilibrium allocation improves the utility of both consumers as opposed to the equilibria without the new commodity. So the equilibrium set is perturbed by the introduction of useless goods. We will see that this is not the case if one considers equilibria with slack.

We could attain the same equilibrium allocation by alternatively introducing a durable good in period 0 which we call paper money. Equally this commodity does not enter the preferences. Give

[1] Polemarchakis and Sciconolfi (1993) avoid this in the incomplete market setting by imposing on the agents to spend their total income.