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Solving the incomplete markets model with aggregate uncertainty using explicit aggregation

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ABSTRACT

We propose a method to solve models with heterogeneous agents and aggregate uncertainty. The law of motion describing aggregate behavior is obtained by explicitly aggregating the individual policy rule. The algorithm is simpler and faster than existing algorithms that rely on parameterization of the cross-sectional distribution and/or a computationally intensive simulation step. Explicit aggregation establishes a link between the individual policy rule and the set of necessary aggregate state variables, an insight that can be helpful in determining what state variables to include in other algorithms as well.

1. Introduction

The behavior of individual agents in DSGE models with aggregate and idiosyncratic risk depends on perceived laws of motions of prices and/or aggregate variables that are, in equilibrium, consistent with the behavior of the individuals. The algorithm developed by Krusell and Smith (1998) finds solutions for parameterized individual policy rules and *separately* parameterized laws of motion for aggregate variables. The individual policy rules describe optimal behavior conditional on the aggregate laws of motions and the aggregate laws of motion provide a close fit for the behavior of the aggregates in a simulated panel that is generated using the individual policy rules. Algan et al. (2008, 2009) and Reiter (2009) parameterize the cross-sectional distribution, which is used to calculate next period's aggregate moments by numerically integrating over the individual choices. These algorithms have in common that (i) an additional function related to an *aggregate* variable, like a moment or the distribution, is separately parameterized and (ii) information about the cross-sectional distribution—obtained by simulating a panel or by parameterizing the distribution—is used to establish a link between the individual and aggregate behavior.

The algorithm developed in this paper establishes the consistency between individual and aggregate behavior in a much more direct manner, namely by explicit aggregation of the individual policy rules. The direct link not only simplifies the calculations considerably, but it is also useful in itself, since it makes clear what information about the aggregate economy should be included in the set of state variables.

To clarify the algorithm we abstract, for the moment, from aggregate and idiosyncratic uncertainty. Consider the following simple model in which all agents are identical except for their initial capital stock. Agents face a standard intertemporal optimization problem taking the return on capital as given. The return on capital is a function of the

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aggregate capital stock only. We parameterize the individual policy function as

$$k' = \Psi_0(s) + \sum_{i=1}^{I} \Psi_i(s) k^i, \tag{1}$$

where s is a vector containing the aggregate state variables. From Eq. (1) it can be seen that the individual policy rule is assumed to be a polynomial in k, but that the dependence of k' on s is not restricted. Using monomials as in Eq. (1) simplifies the exposition. In Section 4, we show that other basis functions such as the elements of orthogonal polynomials or B-splines could be used as well.

A key step in models with heterogeneous agents is to establish a law of motion for aggregate capital, K. Given the expression in Eq. (1), this aggregate law of motion for K follows directly from aggregating the individual policy rule. That is,

$$K' = \Psi_0(s) + \sum_{i=1}^{I} \Psi_i(s) M(i), \tag{2}$$

where K' is next period's per capita capital stock and $M(i)$ is the cross-sectional average of k^i with $K = M(1)$. Note that we need an expression for the average *level* of the capital stock and not, for example, for the average of the log capital stock. Consequently, the left-hand side of Eq. (1) has to be equal the level of k'. In addition, explicit aggregation requires that the coefficients of the monomials, $\Psi_i(s)$, depend only on the aggregate state variables, s, and not on k.

Eq. (2) makes clear that the I cross-sectional moments corresponding to the I monomials, k^i, are required as inputs for predicting K'. That is, the aggregate set of state variables, s, should include $M(1)$ through $M(I)$ and contains, thus, as many aggregate moments as there are basis functions in the approximating individual policy function. But if the first I cross-sectional moments are state variables, then we need aggregate laws of motions to predict these moments as well, since they appear as arguments in next period's policy function. If we had individual policy rules for $(k')^j$, $j = 1, \ldots, I$, then one could get the corresponding aggregate policy rules by explicit aggregation. One way to get a policy rule for $(k')^j$ for $j > 1$ is to use the one that is implied by the approximation for k' given in Eq. (1). This is a polynomial of order $I^j > I$, which means that additional moments would have to be added to s. Then additional policy rules would be needed to predict these additional moments, which in turn would introduce more state variables. Without modification, a solution based on explicit aggregation requires including an infinite number of moments as state variables whenever $I > 1$.

The key approximating step of our algorithm is to break this infinite regress problem and to construct separate approximations to the policy rules for $(k')^j$ by projecting $(k')^j$ on the space of the first I monomials. Thus,

$$(k')^j = \Psi_{(k')^j,0} + \sum_{i=1}^{I} \Psi_{(k')^j,i}(s) k^i, \quad 1 < j \leq I. \tag{3}$$

The coefficients of the approximating functions in (1) and (3) can now be solved with standard projection techniques.

The algorithm does not need a complete characterization of the cross-sectional distribution and, thus, does not have to rely on simulation procedures or a parameterization of the cross-sectional distribution to generate this information. The individual policy rules make clear what aspects of the cross-sectional distribution are needed to construct aggregate laws of motions. Those are the first I moments. By directly approximating the policy rules for $(k')^j$ with $1 \leq j \leq I$ we can get—using the equations of the model and explicit aggregation—a law of motion that describes the joint behavior of these J moments that is consistent with individual behavior. The algorithm, therefore, captures the information about the cross-sectional distribution that is needed to solve the model.

2. Model to solve

First-order and equilibrium conditions: Our numerical solution to the incomplete markets economy with aggregate uncertainty described in Den Haan et al. (2009) consists of individual policy functions, $k'(\varepsilon, k, a, M; \Psi)$, where ε is the (exogenous) individual employment status,[1] k the individual capital stock, a the exogenous aggregate state, Ψ the coefficients of the policy function, and M a set of cross-sectional means of k^j, $1 \leq j \leq I$, measured at the beginning of the period after the new employment status has been observed. We condition the cross-sectional moments on the employment status. As will become clear below, this is a natural thing to do for our algorithm, but it is not necessary. That is, instead of using, for example, the mean capital stocks of the employed and the unemployed, we could use instead just the per capita capital stock.

The standard projection procedure to solve for Ψ consists of the following three steps:

1. Construct a grid of the state variables.

[1] In equations, ε takes on the value 1 when the agent is employed and the value 0 when the agent is unemployed. As a subscript, ε is set equal to e when the agent is employed and equal to u when the agent is unemployed.