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Robust nonexistence of equilibrium with incomplete markets

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Abstract

We construct a pure exchange economy with spot and real security markets for which there does not exist a competitive equilibrium. Moreover, we show that the problem of nonexistence is robust to small perturbations of the endowments of the consumers. The result is driven by a lack of strict convexity of preferences.

JEL classification: D0; D5

Keywords: General equilibrium; Nonexistence; Incomplete markets

1. Introduction

Hart (1975) exhibited an example of an economy with forward and spot markets that does not have a competitive equilibrium. Polemarchakis and Ku (1990) credit Kreps (1979) with first observing the nongeneric nature of this example. Subsequent work by various authors has provided generic existence results, showing that Hart's example was indeed degenerate.[1] The canonical model under consideration in these papers is a two-period economy with one state of the world in the first period and a finite number of states in the second. There are spot markets in each period and state; additionally, there are real asset markets in the first period.[2] Two cases may be differentiated: the "easy" case of as many assets as states

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[1] For an introduction to general equilibrium with incomplete markets, see Geanakoplos (1990).

[2] Models with financial assets have also been considered, e.g. Werner (1985). Since the payout of a financial asset is exogenously fixed, the problems associated with the dependence of asset payoffs on spot market prices do not arise.

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(McManus, 1984; Repullo, 1986; Magill and Shafer, 1990), and the "hard" case of fewer assets than states, addressed by Duffie and Shafer (1985). All these papers assume that the preferences of the consumers are smooth and strictly convex, and they prove that for generic endowments and real asset structures equilibria exist in these models. Furthermore, Magill and Shafer (1990) show that these incomplete market equilibria are allocation equivalent to the contingent market equilibria if the asset structure is "regular", and that for two-period models, assets are generically regular if there are at least as many assets as states.

In this paper, we show using an example that, if preferences are not strictly convex, then for a fixed asset structure the problem of nonexistence of equilibria can be robust to perturbations in the endowments of the consumers. The model we construct has two time periods, with two states of nature in the second period; there are two assets that are traded in the first period. We fix the asset structure and the preferences of the consumers, and vary the endowments of the consumers. It is well known (cf. Geanakoplos, 1990) that if such an economy had an equilibrium price vector at which the monetary returns from the two assets are different, then there exists an equilibrium of the associated Arrow–Debreu economy—i.e. one where all commodity markets open in the first period and there are no asset markets—that achieves the same allocation. In our example, the unique Arrow–Debreu equilibrium prices result in the payoffs from the two assets being the same for an open set of endowments. Thus, if these economies were to have an equilibrium, it would have to be a pure spot market equilibrium. But the spot market equilibria of these economies involve prices where the assets have different monetary returns. This then implies that these economies have no equilibria.

The asset structure here is such that for generic prices the returns from the two assets are different. It follows from Magill and Shafer (1990) that if the preferences of the consumers are smooth and strictly convex, then for generic endowments the equilibria of the incomplete market economy coincide with the equilibria of the Arrow–Debreu economy. The preferences in our model, while strictly monotonic and convex, are, however, not strictly convex. Consequently, there exists an open set of endowments for which the unique Arrow–Debreu equilibrium price vector lies in the nongeneric set where the assets are payoff equivalent.[3]

2. The example

There are two time periods, date 0 and date 1. There is only one state of nature in date 0, while in date 1 there are two, which are denoted states 1 and 2. (For simplicity in notation, we sometimes refer to date 0 as state 0.) In state $i = 0$ and 2, there is only one consumption good, x_i; in state 1, there are two goods, x_1 and y_1. There are two consumers, denoted A and B. Neither consumer cares about consumption in state 0. Their utility functions are given by:

$$U^{\mathrm{A}}(x_1, y_1, x_2) = \min\{2x_1 + y_1, x_1 + 2y_1\} + x_2$$

[3] Robust nonexistence has also been shown by Polemarchakis and Ku (1990) and Momi (2001). The former consider options for which there also exist open sets of endowments such that the asset return matrix drops rank. In response, Bottazzi (1995, 2002) derives conditions/characterizations of assets that have smooth payoffs, thus guaranteeing endowment-generic existence. Momi (2001) considers a production economy with incomplete stock markets and short sales under the Drèze criterion. In this case, it is the production side of the economy that causes equilibrium prices not to vary smoothly with endowment changes.