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Markovian equilibrium in infinite horizon economies with incomplete markets and public policy

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Abstract

We develop an isotone recursive approach to the problem of existence, computation, and characterization of *nonsymmetric* locally Lipschitz continuous (and, therefore, Clarke-differentiable) Markovian equilibrium for a class of infinite horizon multiagent competitive equilibrium models with capital, aggregate risk, public policy, externalities, one sector production, and incomplete markets. The class of models we consider is large, and examples have been studied extensively in the applied literature in public economics, macroeconomics, and financial economics. We provide sufficient conditions that distinguish between economies with isotone Lipschitz Markov equilibrium decision processes (MEDPs) and those that have only locally Lipschitz (but not necessarily isotone) MEDPs. As our fixed point operators are based upon order continuous and compact nonlinear operators, we are able to provide sufficient conditions under which isotone iterative fixed point constructions converge to extremal MEDPs via successive approximation. We develop a first application of a new method for computing MEDPs in a system of Euler inequalities using isotone fixed point theory even when MEDPs are not necessarily isotone. The method is a special case of a more general mixed monotone recursive approach. We show MEDPs are unique only under very restrictive conditions.

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Finally, we prove monotone comparison theorems in Veinott's strong set order on the space of public policy parameters and distorted production functions.

Keywords: Markovian equilibrium; Monotone methods; Lattice Programming

1. Introduction

Over the last two decades researchers in macroeconomics and financial economics have proposed a number of new theoretical frameworks in an effort to assess the role of behavioral heterogeneity and incomplete markets on both the positive and the normative aspects of equilibrium economic fluctuations. Such models have proven to be important for researchers studying the interplay between public policy, incomplete markets, income risk (both aggregate and idiosyncratic), asset pricing, production uncertainty, quantitative welfare assessments of the costs of business cycles, financial economies, and macroeconomics. With a few notable exceptions, a majority of this work has focused on numerical characterizations of Markovian equilibrium. In addition even when theoretical work on Markovian equilibrium has been done, there has been little relationship provided to connect theoretical constructions that have used primarily pure topological fixed point methods to the study of numerical solutions for these models. This paper makes a significant contribution by bridging this large gap between theory and numerical applications for an important class of equilibrium models of economic fluctuations with behavioral heterogeneity, incomplete markets, public policy, and bounded production nonconvexities. In doing so, we suggest the possibility of broader applications of monotone methods to more complicated dynamic economies.

The problems associated with the tractability of heterogeneous agent economies (both theoretical and applied) from a mathematical perspective are well-known. One key conceptual question concerns the structure (or existence) of Markovian equilibrium in a multi-asset model. In models with many assets, little is known about the Markovian equilibrium with incomplete markets where the second welfare theorem fails, production is nonconvex, and there is public policy.[1] In the single-asset incomplete markets model of Bewley (1986), there have been two versions proposed. On the one hand, an important version of the Bewley model is presented in the work of Aiyagari (1994), Krusell and Smith (1998), and Miao (2002). A key feature of this class of single-asset models is that agents face a *continuum* of uninsured (agent specific) income risks without facing an aggregate risk. In some versions

[1] One exception is Kubler and Schmedders (2003) where the issue of the existence of stationary Markov equilibrium in simple multi-asset incomplete markets model with collateral constraints is addressed. Their paper suggests the possibility of developing a general isotone iterative method based on "set-to-set" maps for computing Markov equilibrium. We remark first their methods do not work in the present single-asset context.

Reffett (2005a) shows that the Kubler–Schmedder's approach is a special case of a more general monotone-map method based on interval condensing methods. Numerical methods for computing such set-to-set maps in principle can be based upon the methods of Rockafellar and Wets (1997).