Available online at www.sciencedirect.com
SCIENCE DIRECT®
Journal of Banking & Finance 28 (2004) 1–17
Journal of BANKING & FINANCE
www.elsevier.com/locate/econbase

Hedging with forwards and puts in complete and incomplete markets ☆

Simon Z. Benninga [a,b], Casper M. Oosterhof [b,*]

[a] *Leon Recanati School of Business Administration, Tel Aviv University, P.O. Box 39010, Ramat Aviv, Tel Aviv 69978, Israel*
[b] *Department of Finance, University of Groningen, P.O. Box 800, 9700 AV Groningen, The Netherlands*

Abstract

We derive general conditions for forward and/or put unbiasedness and show that restrictions on the probability distribution suffice for simultaneous unbiasedness of forwards and puts, even if consumers are assumed to be risk averse. We examine the optimal production and hedging decisions by a risk-averse producer. If the producer's state prices are derived from his marginal rates of substitution, an unbiased market forward price is perceived as overpriced and an unbiased market put price as underpriced. Even in this case the full hedging and separation theorems still hold and, contrary to previous literature, there is a hedging role for puts.

JEL classification: D21; D81; G10
Keywords: Hedging; Put options; Forward contracts; Unbiasedness

1. Introduction

Income uncertainty and optimal hedging decisions by a competitive commodity producer have been the object of considerable research. This paper examines two issues which are not addressed or have caused some confusion in the hedging literature. We first derive general conditions under which forward and/or put price

☆ This paper was presented at the X International Tor Vergata Conference on Banking and Finance held in Roma on 5/7 Dec. 2001.

* Corresponding author.
E-mail addresses: benninga@post.tau.ac.il (S.Z. Benninga), c.m.oosterhof@eco.rug.nl (C.M. Oosterhof).

0378-4266/$ - see front matter
doi:10.1016/S0378-4266(02)00390-4

unbiasedness occurs. Contrary to the traditional belief that unbiasedness occurs only under risk-neutrality, we show that restrictions on the probability distribution suffice for unbiasedness, even if consumers are assumed to be strictly risk averse. Second, we examine the optimal production and hedging decisions by a risk-averse producer. Hedging is utility-enhancing for this producer only if his private state prices (derived from the marginal rates of substitution) differ from the market state prices. If the producer's state prices are derived from his marginal rates of substitution, he will perceive an unbiased market forward contract to be overpriced and an unbiased market put price to be underpriced. Contrary to the previous literature we show there is a hedging role for put options together with forward contracts.

In a pioneering article, McKinnon (1967) presents a model of a commodity producer who minimizes income volatility in a mean–variance framework. He shows that the correlation between stochastic price and production is crucial in the optimal hedging decision. A missing feature in McKinnon's model is that production cannot be chosen. Baron (1970) and Sandmo (1971) develop a model of optimal production under price uncertainty, which is extended by Danthine (1978), Holthausen (1979) and Feder et al. (1980) to incorporate optimal hedging decisions as well. They show that, when output is non-random, the well-known separation theorem holds. The optimal production decision is independent of the producer's risk preferences and expectations and can be separated from the optimal hedging decision. If the forward price is unbiased, the optimal production decision is to produce until the marginal costs equal the forward price and the optimal forward position is a full hedge.

The results above – extended by Benninga et al. (1983) and Lapan et al. (1991) – apply to a competitive producer who faces price risk only. For most commodities, however, a producer faces multiple sources of risk. Lapan and Moschini (1994) consider a producer facing price, production, and basis risk. They derive an exact solution to the optimal hedging problem under the assumption that price, production and basis risk are joint-normally distributed and that the producer maximizes an exponential utility function. An important finding is that the optimal hedge depends on the degree of risk aversion, even if the forward price is assumed to be unbiased.

The use of options as a hedging instrument has been examined much less than the use of futures. Lapan et al. (1991) consider a producer facing price and basis risk and compare the use of futures to put options as a hedging device. They show that, when the futures price is unbiased, options are redundant hedging instruments since futures provide a payoff that is linear in price risk. Moschini and Lapan (1995) study the problem of a producer facing price, (non-linear) basis, and production risk. They provide analytical solutions to the use of futures contracts and straddles, assuming an exponential utility function and joint–normal distributions between the risk factors. Under the assumption of unbiased forward and straddle prices, they show that the optimal strategy is to buy straddles along with a short position in futures. Battermann et al. (2000) compare the use of forward contracts and put options within a one period utility framework. They show that, in case of unbiased put prices, the optimal hedging strategy is to overhedge and the optimal output decision is to produce up to a point where the marginal costs are less than the forward price (assuming