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Core concepts for incomplete market economies

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ABSTRACT

We examine the notion of the core when cooperation takes place in a setting with time and uncertainty. We do so in a two-period general equilibrium setting with incomplete markets. Market incompleteness implies that players cannot make all possible binding commitments regarding their actions at different date-events. We unify various treatments of dynamic core concepts existing in the literature. This results in definitions of the Classical Core, the Segregated Core, the Two-stage Core, the Strong Sequential Core, and the Weak Sequential Core. Except for the Classical Core, all these concepts can be defined by requiring the absence of blocking in period 0 and at any date-event in period 1. The concepts only differ with respect to the notion of blocking in period 0. To evaluate these concepts, we study three market structures in detail: strongly complete markets, incomplete markets in finance economies, and incomplete markets in settings with multiple commodities. Even when markets are strongly complete, the Classical Core is argued not to be an appropriate concept. For the general case of incomplete markets, the Weak Sequential Core is the only concept that does not suffer from major defects.

1. Introduction

We examine the notion of the core in the standard two-period general equilibrium model with incomplete markets. Market incompleteness implies that players cannot make all possible binding commitments regarding their actions at different date-events. In the literature, a number of proposals can be found for the appropriate notion of the core in a context with restricted commitment possibilities. Many of these contributions were developed independently, and in environments as distinct as economies with incomplete markets, economies with transaction costs, dynamic monetary economies, deterministic capital accumulation models, and sequences of transferable utility games.

We unify the various treatments of dynamic core concepts that so far are scattered around in the literature, and find that several of the concepts proposed actually coincide. This results in definitions of the Classical Core, the Segregated Core (Grossman, 1977; Bester, 1984; Repullo, 1988a), the Two-stage Core (Koutsougeras, 1998), the Strong Sequential Core (Gale, 1978; Becker and Chakrabarti, 1995; Predtetchinski et al., 2002; Kranich et al., 2005), and the Weak Sequential Core (Kranich et al., 2005; Predtetchinski et al., 2006). Except for the Classical Core, all these concepts can be defined by requiring the absence of blocking in period 0 and at any date-event in period 1. The concepts only differ with respect to the notion of blocking in period 0.

Consider a particular allocation and portfolio plan. Since the only commitment possibilities are those implied by the portfolio plan, a coalition can block at a date-event in period 1 if it can redistribute its initial endowments and proceeds from the portfolio plan in such a way as to make every coalition member better off. All the core concepts, with the exception of the Classical Core, agree with this notion of blocking. The Classical Core is essentially a static concept and ignores the option of blocking at a date-event in period 1.

To assess whether a coalition blocks in period 0, it has to evaluate the consequences of a deviation regarding consumption in period 1. It is here that the various concepts differ. In the Segregated Core, it is assumed that net trades in period 1 are not affected by a deviation in period 0. The Two-stage Core takes a very conservative point of view in that coalition members are only guaranteed their initial endowments plus the proceeds from their asset portfolio. The Strong Sequential Core agrees with the Classical Core in that, it regards any future redistribution of endowments as feasible. Since, contrary to the Classical Core, the Strong Sequential Core allows for blocking in period 1, it is a refinement of the Classical Core. For the Weak Sequential Core, it is assumed that coalition members can coordinate on a particular element of the core of the ex-post economies in period 1 that result after a deviation.

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We evaluate these core concepts for three different market structures: strongly complete markets, incomplete markets in finance economies, and incomplete markets in settings with multiple commodities. Markets are said to be strongly complete if every consumption bundle can be implemented today by means of the existing assets. Finance economies are economies in which contingent on each date-event, there is exactly one commodity being traded. For finance economies we do not impose assumptions on the market structure. Finally, we study the multiple commodity case with a general market structure.

One may expect that when markets are strongly complete all core concepts coincide. However, such is not the case. The only two concepts that coincide are the Two-stage Core and the Strong Sequential Core. Both cores are contained in the Weak Sequential Core and the Classical Core, but there is no general relationship between the latter two. The Segregated Core does not satisfy any general relationship with any of the other concepts. We argue that the Classical Core is not restrictive enough for dynamic economies with strongly complete markets, as it does not take into account new blocking opportunities that arise in the future. The Classical Core is therefore not an appropriate concept to study dynamic economies. The Segregated Core on the other hand is too permissive, as it may even include allocations that fail to be individually rational, which also discards the Segregated Core as a reasonable concept. When we impose some additional assumptions, in particular the assumption that the Classical Core of relevant ex-post economies is non-empty and the assumption that Strong and Weak Pareto Optimal allocations coincide, we can show that all core concepts coincide with the exception of the Segregated Core, which is shown to contain the other concepts.

In finance economies, i.e. economies where one commodity per date-event is being traded, and a general market structure, it is still true that the Two-stage Core and the Weak Sequential Core coincide, and for finance economies these two concepts even coincide with the Segregated Core. The equivalence with the Classical Core and the Strong Sequential Core is now lost, due to the potential market incompleteness. The Strong Sequential Core is a proper subset of all the other concepts, whereas apart from the relation to the Strong Sequential Core, the Classical Core does not satisfy other relationships. In the extreme case of finance economies without asset markets, the Strong Sequential Core is typically empty, the Classical Core includes some Pareto efficient allocation, and the other concepts coincide with the initial endowments, the only reasonable prediction in this case. It follows that the Strong Sequential Core is not an appropriate concept when studying economies with incomplete markets.

In the general case – multiple commodities and potentially incomplete asset markets – we show that competitive equilibria belong to the Segregated Core and the Two-stage Core. In general it is not true that competitive equilibria belong to the Classical Core, the Strong Sequential Core, and the Weak Sequential Core. This is an indication that the Segregated Core and the Two-stage Core are too permissive. The constrained suboptimality results of Geanakoplos and Polemarchakis (1986) state that competitive equilibria are not constrained optimal, so can typically be improved upon while only making use of the existing assets in the economy. It is then only natural that competitive equilibria typically do not belong to an appropriate concept of a dynamic core. We are left with the Weak Sequential Core as the only concept that does not suffer from major deficiencies. We show that in the general case, the Strong Sequential Core is a subset of the Classical Core and the Weak Sequential Core, and that the Weak Sequential Core is a subset of the Two-stage Core. Examples illustrate that there are no further relationships.

The outline of the paper is as follows. We specify the model in Section 2 and give the formal definitions of the various core concepts in Section 3. We compare these concepts for the case with strongly complete markets in Section 4. The one-commodity case is studied in Section 5. Section 6 examines the relation of the core concepts and the competitive equilibrium. We discuss the general case with incomplete markets and multiple commodities in Section 7. Section 8 concludes.

2. The model

Consider an economy with two time-periods, $t \in \{0, 1\}$. In time-period 1 trade takes place conditional on the occurrence of a date-event s in the finite set of date-events S. We define the date-event for time-period 0 as $s = 0$, so the set of all date-events is $S' = \{0\} \cup S$. At each date-event there is trade in a finite set L of non-durable consumption goods.

There is a finite number of households $h \in H$ who participate in the economy. Household h has initial endowments $e^h = (e^h_s)_{s\in S'} \in \mathbb{R}^{S'L}$. The profile of initial endowments is $e = (e^h)_{h\in H}$. The preferences of household h are represented by its utility function $u^h : X^h \to \mathbb{R}$, with the consumption set X^h a subset of the commodity space $\mathbb{R}^{S'L}$. We denote $\prod_{h\in H} X^h$ by X, with typical element x. Let $\mathcal{C}$ be the collection of all coalitions, i.e. the collection of all non-empty subsets of H. For $C \in \mathcal{C}$, we denote $\prod_{h\in C} X^h$ by X^C, with typical element x^C.

For $\bar{s} \in S'$, we denote the consumption $(x^h_s)_{s\in S'\setminus\{\bar{s}\}}$ of a household h outside date-event $\bar{s}$ by $x^h_{-\bar{s}}$. The utility function u^h is *locally non-satiated* in date-event $\bar{s} \in S'$ if for every $\bar{x}^h \in X^h$ and for every $\varepsilon > 0$ there is $x^h \in X^h$ with $x^h_{-\bar{s}} = \bar{x}^h_{-\bar{s}}$ such that $\|x^h_{\bar{s}} - \bar{x}^h_{\bar{s}}\|_\infty < \varepsilon$ and $u^h(x^h) > u^h(\bar{x}^h)$.

For $\bar{x}^h_0 \in \mathbb{R}^L$ we define the set $X^h(\bar{x}^h_0) = \{x^h \in X^h \mid x^h_0 = \bar{x}^h_0\}$ as the set of feasible consumption bundles with state 0 consumption equal to $\bar{x}^h_0$. The consumption set X^h is said to be *state separable* if for every $x^h_0 \in \mathbb{R}^L$ the set $X^h(x^h_0)$ is either empty or it has the product form $\{x^h_0\} \times \prod_{s\in S} X^h_s(x^h_0)$, where $X^h_s(x^h_0)$ is a subset of $\mathbb{R}^L$. For state separable X^h, we define the set

$$X^h_{0,s} = \cup_{x^h_0\in\mathbb{R}^L}\{x^h_0\} \times X^h_s(x^h_0), \quad s \in S,$$

with the convention that a product involving an empty set is empty itself. The utility function u^h is said to be *state separable* if there exist functions $u^h_s : X^h_{0,s} \to \mathbb{R}$, $s \in S$, such that $u^h(x^h) = \sum_{s\in S} u^h_s(x^h_0, x^h_s)$.

We apply the following assumption throughout the paper.

Assumption 2.1. For $h \in H, X^h$ is non-empty, closed, convex, and state separable, and the utility function is continuous, state separable, and locally non-satiated in every date-event.[1]

Von Neumann–Morgenstern utility functions would be a prominent example of utility functions satisfying Assumption 2.1. State separability is a natural requirement since only one out of the future states of nature materializes.

At date 0, there is a finite set J of assets. An asset $j \in J$ pays a dividend $d_{sj} \in \mathbb{R}^L$ at date-event $s \in S$. We denote the $(L \times J)$-matrix of dividends by $D_s = (d_{sj})_{j\in J}$ and the $(SL \times J)$-asset payoff matrix by $A = (D_s)_{s\in S}$. We assume that assets are in zero net supply. At date-event 0, household h chooses a portfolio holding $\theta^h \in \mathbb{R}^J$ and a consumption bundle $x^h_0 \in \mathbb{R}^L$. Households choose a consumption bundle x^h_s conditional on s at date-events in S. The only commitments households can make regarding the future are those implied by their portfolio holding θ^h. We denote $\prod_{h\in H} \mathbb{R}^J$ by

[1] Most of our results do not rely on X^h being non-empty, closed, and convex. We merely make these assumptions to rule out pathological cases.