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Default and exogenous collateral in incomplete markets with a continuum of states☆

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Abstract

We consider an economy with a finite number of agents, a continuum of states and incomplete markets where individuals do not keep their promises contrary to the Arrow–Debreu model of general equilibrium with complete markets (GE) and the general equilibrium model with incomplete markets (GEI) where they do keep them by assumption. We show that in addition to usual assumptions on utilities and endowments the equilibrium for this economy must exist if sales of assets are backed by some nonzero collateral. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Incomplete markets; Asset promises; Collateral; Durability

1. Introduction

In the Arrow–Debreu model of general equilibrium with complete markets (GE) and likewise in the general equilibrium model with incomplete markets (GEI) agents keep all their promises by assumption. It is well known that in the latter, without the lower bound on short sales, the equilibrium can fail to exist, see Hart (1975). However, Duffie and Shafer (1985) prove that Hart's example is held only on a null set of parameters, i.e. the equilibrium exists generically. Later Dubey and Geanakoplos (1989, 1990) extend the GEI model to another where agents are not obligated to keep promises and they prove that the equilibrium always exists for this model whenever there exist a lower bound on short sales. As always this bound is necessary to guarantee the existence.

On the other hand, when these models were extended for a continuum of states — see Mas-Colell and Monteiro (1996), Hellwig (1996), Mas-Colell and Zame (1996) and Monteiro (1995, 1996) — in addition to the hypothesis of the lower bound short on sales it is

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required that ex post endowments be non-negative. This assumption is much stronger but indispensable, see Mas-Colell and Zame (1996) and Monteiro (1996). Araújo et al. (1997) eliminated this requirement by introducing default subject to utility penalties, in the set-up of Dubey and Geanakoplos (1989).

The purpose of this paper is to show the existence of equilibria in an economy with a continuum of states and incomplete markets without imposing any assumption on ex post endowments. This awkward assumption is eliminated by introducing default subject to sale of assets that are backed by the purchase of durable goods serving as collateral. We extend, for a continuum of states, the pioneering study of collateral by Geanakoplos et al. (1995), where results on existence of equilibria are provided, for exogenously given collateral coefficients. This extension is carried out modeling the second period uncertainty by a probability space without atoms. Specifically, we consider $S \equiv [0, 1]$ as the set of states of nature equipped by the sigma algebra of Lebesgue and of course the measure of Lebesgue. Moreover, we make some simplifications which will not alter the arguments used in the proof on existence. For instance, we allow for each debtor (seller of assets) to hold the full collateral that secures the sale of asset, we also assume that the collateral, as well as any consumption good is durable. This durability is measured by a linear transformation depending on the state of nature that has been solved in the second period.

The existence argument uses a finite-dimensional approximation and applies Fatou's lemma to a uniformly bounded sequence of equilibrium variables of the truncated economies. This mathematical approach of existence of equilibria with a continuum of states was already used in the non-default model of Hellwig (1996) and Monteiro (1996) as well as in the default model of Araújo et al. (1997, 1998). The proof of existence can be described intuitively as follows: First, we define the two-period truncated economy which is obtained by partitioning $S \equiv [0, 1]$ in a finite number of sub-intervals so that this new economy has a finite number of state of nature, which is solved in the second period, equal to number of sub-interval in which is partitioned S. Next, we appeal to the existing result for a finite state space, by Geanakoplos et al. (1995), to guarantee existence in economies with truncated state spaces and then use a multi-dimensional version of Fatou's lemma to infer existence for the continuum economy. Several lemmas show that the sequences of Lagrange's multipliers and equilibrium allocations are uniformly bounded as required for the application of Fatou's lemma.

Although this technique is not original, the intuition behind the success of its use here is interesting. Short sales allocations are uniformly bounded because these sales must be backed by the purchase of durable goods serving as collateral. Lagrange's multipliers are uniformly bounded because second-period ex post endowments (including asset real returns and depreciated first-period consumption) always exceed ex ante endowments in the collateral model. Under an assumption that bounds the latter uniformly away from zero it becomes easy to show the marginal utilities of income in the second period must be uniformly bounded from above.

Notice that the only condition to guarantee the existence of the equilibrium, in addition to usual assumptions on utilities and endowments, is that the sales of assets should be backed by durable physical commodities (collateral). Essentially what happens is that this assumption helps to find a lower bound on short sales since the collateral is fixed. Collateral has the advantage that the lender need not bother with the reliability or even the identity of the

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