



Excess demand function around critical prices in incomplete markets

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ABSTRACT

We show that the aggregate excess demand function in an economy with incomplete real asset markets can be characterized by Walras' law, homogeneity, and continuity around critical prices that cause one-dimensional drop of the dimension of the budget set.

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1. Introduction

Sonnenschein (1973), Mantel (1974) and Debreu (1974) proved that the aggregate excess demand function in the Arrow–Debreu exchange economy is characterized on any compact set of prices by continuity, homogeneity, and Walras' law. This research has been extended to incomplete market models, and similar results have been obtained by Bottazzi and Hens (1996), Gottardi and Hens (1999), Chiappori and Ekeland (1999, 2000), Gottardi and Mas-Colell (2000) and Momi (2003). In these papers, the characterization of the aggregate excess demand function is obtained, at most, on a compact set of prices over which the dimension of the budget set is constant.

When we compare the Arrow–Debreu model and the incomplete market model, one of the most prominent differences is that the (excess) demand function in the latter could be discontinuous at critical prices where the budget set drops its dimension. This discontinuity causes great difficulties in dealing with the incomplete market model. For example, as demonstrated in Hart's (1975) example, this discontinuity prevents general existence of the equilibrium in the incomplete market model. Although the set of critical prices is negligible in its size and the exceptional excess demands at exactly such prices might be of little interest, the behavior of the aggregate excess demand function around such prices would be worth considering. The lack of the characterization of the aggregate excess demand around the critical prices in the incomplete market model is, clearly, a drawback as compared to the Sonnenschein–Mantel–Debreu Theorem in the Arrow–Debreu economy.

The purpose of this paper is to investigate the characterization of the aggregate excess demand function around the critical prices. We prove that in an economy with real asset markets, on any compact price set where the dimensional drop of the budget set is at most one-dimensional, continuity, homogeneity, and Walras' law still characterize the aggregate excess

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demand function except the discontinuous values at the critical prices. We prove the result by extending the argument of Momi (2003).

As an important application of the result, we also show that if the dimensional drop of the budget set is at most one-dimensional on the whole price set, then any compact set of the budget sets could be the set of the equilibrium budget sets of an incomplete market economy. This is parallel to Mas-Colell (1997) who refined the argument of Debreu (1974) and proved that any compact price set could be the equilibrium price set of an Arrow–Debreu economy.

Clearly, a shortcoming of our approach is that it cannot be applied when the budget set decreases its dimension more than two dimensions. Whether we could characterize the aggregate excess demand function around such critical prices is an interesting open question beyond our scope. Examining why our approach could not be applied in this case might provide a hint to deal with this problem.

Section 2 describes the model, which is a standard general equilibrium model with incomplete real asset markets. Section 3 discusses the difficulty in dealing with the excess demand function around critical prices. Section 4 reviews the properties of the aggregate excess demand function in the incomplete market economy populated by consumers with standard preference orderings. Section 5 shows our results: the aggregate excess demand function characterization and the equilibrium budget set characterization. Proofs of the results are provided in Section 6.

2. The model

We consider a standard two-period exchange economy with incomplete real asset markets. There are S possible states in the second period and N goods in each state, so that R^M , where $M = (S + 1)N$ with the first period as state 0, represents the total commodity space. There are $J(\leq S)$ real assets $V^j, j = 1, \dots, J$, each of which promises the delivery of a bundle of commodities $V_s^j = (V_{s1}^j, \dots, V_{sN}^j)$ if state $s \in \{1, \dots, S\}$ occurs in the second period. The budget set, which the excess demand vectors $z = (z_0, \dots, z_S) \in R^M$ have to satisfy, is then represented as

$$L(p) = \left\{ z \in R^M \mid \begin{array}{l} pz = 0, \\ p_s z_s = \sum_j p_s V_s^j \theta^j, s = 1, \dots, S, \theta = (\theta^1, \dots, \theta^J) \in R^J \end{array} \right\},$$

where $p = (p_0, p_1, \dots, p_S) \in \Delta = \{p \in R_{++}^M \mid \|p\| = 1\}$ represents a normalized present value price system.¹ We call the $S \times J$ matrix $V(p) \equiv (p_s V_s^j)_{s=1, \dots, S}^{j=1, \dots, J}$ as a payoff matrix. Without loss of generality, we assume that the assets are not redundant. Note that $L(p)$ is a $k \equiv M - (S - J) - 1$ -dimensional linear subspace in R^M if and only if $V(p)$ is of full rank. We call such a price “good” and write Δ^g to denote the set of good prices: $\Delta^g = \{p \in \Delta \mid \text{rank} V(p) = J\}$. We call the critical price where the rank of $V(p)$ is less than J (that is, where the dimension of $L(p)$ is less than k) as “bad” and define $\Delta^b = \Delta \setminus \Delta^g = \{p \in \Delta \mid \text{rank} V(p) < J\}$. We let $G^k(R^M)$ denote the set of k -dimensional linear subspaces in R^M called Grassmann manifold. It is well known that the Grassmann manifold $G^k(R^M)$ is metrizable. We let $\tilde{d}(L, L')$ denote the distance between L and L' in $G^k(R^M)$ with respect to the introduced metric. We define $G_{++}^k(R^M) = \{L \in G^k(R^M) \mid L \cap R_{++}^M = \emptyset\}$. For a good price $p \in \Delta^g$, $L(p)$ is, of course, an element of $G_{++}^k(R^M)$.

A consumer indexed by i is represented by (\tilde{z}^i, ω^i) where \tilde{z}^i denotes her strictly convex, monotone, continuous, complete preference ordering on the consumption set R_{++}^M and $\omega^i \in R_{++}^M$ denotes her initial endowment. The excess demand of consumer i at price p is, hence, defined as $z^i(p) = \{x - \omega^i \in R^M \mid (x - \omega^i) \in L(p) \text{ and } x' \tilde{z}^i x \text{ for any } x' \in R_{++}^M \text{ satisfying } (x' - \omega^i) \in L(p)\}$.

3. Convergence of budget sets around a critical price

In this section, we discuss the difficulties in dealing with the excess demand function around critical prices.

Let two sequences of good prices $\{p'_n\}$ and $\{p''_n\}$ converge to a good price $p \in \Delta^g$: $p'_n \rightarrow p$ and $p''_n \rightarrow p$ as $n \rightarrow \infty$. Then, the corresponding budget sets $L(p'_n)$ and $L(p''_n)$ clearly converge to the budget set $L(p)$. That is, the function $p \rightarrow L(p)$ is continuous around a good price. If p' and p'' are in a neighborhood of a good price p , then $L(p')$ and $L(p'')$ are in a neighborhood of $L(p)$. Then, the approach of Bottazzi and Hens (1996) or Chiappori and Ekeland (1999) can be applied to achieve a local characterization of the excess demand function around a good price p .

The situation is, however, quite different when $\{p'_n\}$ and $\{p''_n\}$ converge to a bad price $\bar{p} \in \Delta^b$. When they converge to a bad price, the prices, of course, converge to each other. However, the corresponding budget sets do not converge to each other. Therefore, the problem is not “local”. In this section, we observe this using a simple example. See Appendix A for a general argument. We consider the simplest example with three future states ($S = 3$), two assets ($J = 2$) with positive payoffs, and

¹ $\|\cdot\|$ denotes the Euclid norm.

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