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Solving the incomplete market model with aggregate uncertainty using a perturbation method

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ABSTRACT

We use a perturbation method to solve the incomplete markets model with aggregate uncertainty described in den Haan et al. [Computational suite of models with heterogeneous agents: incomplete markets and model uncertainty. Journal of Economic Dynamics & Control, this issue]. To apply that method, we use a "barrier method" to replace the original problem with occasionally binding inequality constraints by one with only equality constraints. We replace the structure with a continuum of agents by a setting in which a single infinitesimal agent faces prices generated by a representative-agent economy. We also solve a model variant with a large (but finite) number of agents. Our perturbation-based method is much simpler and faster than other methods.

1. Introduction

This paper explains how a perturbation method can be applied to solve the incomplete markets model with aggregate uncertainty described in den Haan et al. (2009). We face two main challenges in applying existing perturbation algorithms to this model: how to deal with the occasionally binding non-negativity constraint for capital and with the continuum of agents. In Sections 2 and 3, we explain how we meet these challenges by using a barrier method, and by replacing the structure with a continuum of agents by a setting in which a single infinitesimal agent faces prices generated by a representative-agent economy; we also discuss a model variant with a large (but finite) number of agents.[1] Note that the simulation results reported in the comparison paper, den Haan (2009), are based on the representative-agent setup. Sections 4 and 5 summarize key properties of the model solution, including policy functions and Euler equation errors. Section 6 concludes the paper.

2. Two challenges for perturbation

2.1. Continuum of agents

The model is a production economy with a continuum of households of unit mass. Currently available "general purpose" computer programs for perturbation analysis are not designed to deal with an infinite number of

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[1] Kim et al. (2004) uses the same approach to solve a heterogeneous agent incomplete markets model with trade in bonds (instead of trade in physical capital).

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variables.[2] In order to apply the perturbation method to this model, we make use of the property that each infinitesimal agent is a price taker. We solve a representative agent version of the economy to generate a process for the wage rate and the rental rate of capital. That is, we express the wage and rental rates as functions of aggregate shocks and the aggregate capital stock only, while ignoring the wealth distribution. We then solve the decision problem of an infinitesimal agent who faces that process.[3]

The assumption that factor prices are generated by a representative-agent economy greatly simplifies the application of the perturbation method, but that assumption is not indispensable. We also consider a model variant in which the continuum of agents is replaced by a large (but finite) number of agents, and in which individual decisions and factor prices are jointly solved for. We find that, as the number of agents rises, individual policy functions in that N-agent model approach those in the "representative-agent" setup.

In closely related research on a heterogenous agent economy, Preston and Roca (2007) explicitly include the second moments of the wealth distribution as a perturbation variable, and solve the model using a second-order accurate perturbation method. Therefore, the wealth distribution in their solution is consistent with individual behavior. However, given the second-order nature of the wealth distribution terms, ignoring the wealth distribution would not affect the results when the model is approximated up to the first order, which is in fact the approach taken in most of this paper.[4]

In another closely related contribution, Reiter (2009) seeks to overcome the local nature of the perturbation method by combining that method with a projection method. Specifically, he solves a model variant with only idiosyncratic shocks by a projection method, and then perturbs its solution with respect to aggregate shocks (up to the first order in his application).

Compared to these contributions, our approach provides the simplest way to apply a perturbation method to the model with heterogeneous agents. The method here is also very fast, especially when compared to projection methods; for example, computing policy functions takes less than a second.[5]

2.2. Inequality constraint

The other challenge is how to deal with the non-negativity constraint for individual capital stocks. Perturbation methods cannot directly be applied to models with occasionally binding inequality constraints. One possible way to deal with this problem is to modify the utility function so that agents are penalized when capital holdings move close to the "barrier" of the zero bound. This "barrier method" (see Luenberger, 1973; Judd, 1998) converts the model with inequality constraints into an optimization problem with only equality constraints, which allows us to apply a standard perturbation method.[6]

Specifically, we consider the following modified utility function:

$$U(c_t^i, k_t^i) = \frac{(c_t^i)^{1-\gamma} - 1}{1-\gamma} + \phi\left[\log\left(\frac{k_t^i}{\bar{k}}\right) - \frac{k_t^i - \bar{k}}{\bar{k}}\right], \tag{1}$$

where c_t^i, k_t^i are an individual household's consumption in period t, and her capital stock at the beginning of that period, respectively. $\phi > 0$ is a coefficient (referred to as a "barrier parameter"), and $\bar{k}$ is the value of the individual capital stock in the (deterministic) steady state of the economy. Due to the term $\log k_t^i$ in the (modified) utility function, the marginal utility of holding capital goes to infinity when k_t^i goes towards zero. This specification ensures that the steady state is independent of ϕ. (Note that this modified utility function penalizes not only capital holdings below the steady state but also capital holdings above the steady state.)

We next describe the model equations used for our computational approach.

3. The model

The budget constraint of an individual household is given by

$$c_t^i + k_{t+1}^i = r_t k_t^i + (1-\tau_t) w_t \bar{l} \varepsilon_t^i + \mu w_t (1 - \varepsilon_t^i) + (1-\delta) k_t^i, \tag{2}$$

where $\varepsilon_t^i \in \{0, 1\}$ is the employment status of the individual; $\bar{l}$ is the time endowment of the household (set at 1/0.9); r_t and w_t are the rental rate of capital and the wage rate, respectively; τ_t is the labor tax rate; μw_t is the unemployment benefit;

[2] See algorithms such as gensys, gensys2, Dynare, PerturbationAIM, and Judd-Jin's PSD. For a detailed comparison of these methods, see Kim et al. (2008).

[3] When solving a heterogeneous-agent model, Krusell and Smith (1998) also assume that households believe that they face a simple stochastic process for aggregate capital and its price. These authors impose the consistency condition that the process for aggregate prices faced by households must be the same as the one that they would estimate from a simulation of the model. The representative-agent structure in this paper does not have such a consistency requirement.

[4] In the economy considered in Preston and Roca (2007), the distribution of capital does not significantly affect aggregate dynamics.

[5] Participating algorithms based on projection methods take between 7 and 2739 min to solve the model. See Table 3 in den Haan's (2009) comparison paper.

[6] Preston and Roca (2007) also use a barrier method.