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Wealth optimization in an incomplete market driven by a jump-diffusion process

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Abstract

We consider a small investor who operates within an incomplete market driven by a diffusion with jumps. His behavior facing the market risks is defined by a utility function. We prove the existence of an optimal portfolio and we characterize this portfolio. The optimal strategy determines a unique equivalent martingale measure which is identified with the Davis' probability. We find explicit formulae for the optimal portfolio, the wealth and the value function in some particular examples of utility functions. Moreover, we prove that the range of the viable prices is reduced by the use of a utility function. At last, we compare the value function with the value function evalued in a market driven by a continuous asset. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Equivalent martingale measure; Optimization problem; Portfolio; Utility function

1. Introduction

Many authors have studied the wealth optimization problem. We can quote, for instance, Merton's works (1971) or Karatzas (1989) written in the case of complete markets, and those of Karatzas et al. (1991), or of Kramkov and Schachermayer (1998) in the case of incomplete markets.

When markets are complete, the existence of optimal strategies and their explicit computations can be found in (Merton, 1971) or in (Jeanblanc and Pontier, 1990), for example. In the Merton's article, the risky assets are continuous while in the one of Jeanblanc and Pontier, the dynamics of the prices are driven by Brownian motion and Poisson processes.

Our work extends those studies and analyses the wealth optimization when the market is incomplete and driven by discontinuous prices. The motivation of this study is the following:

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in such a market, there are several equivalent martingale measures and each one of them defines a price compatible with no arbitrage. The search of criterion in order to reduce the range of prices is essential for an investor. We prove that the use of a utility function is a well-adapted answer to this problem.

We suppose that the market is defined by a bond and a risky asset whose dynamics are solutions of the following equation:

$$\mathrm{d}S_t = S_{t-}\left[b(t)\,\mathrm{d}t + \sigma(t)\,\mathrm{d}W_t + \sum_{i=1}^{i=k}\varphi_i(t)\,\mathrm{d}M_{i,t}\right],$$

where W is a Brownian motion and M_i, $i = 1, \dots, k$, are the compensated martingales of Poisson processes N_i. We assume that the processes W and N_i, $i = 1, \dots, k$, are independent.

We consider an economic agent whose behavior facing the risk is determined by a utility function U. He invests his wealth in the two assets and wants to maximize the expected utility of wealth at time T. We define the value function V as

$$V(t, x) = \sup_{\pi \in \mathcal{A}(t,x)} E(U(X_T^\pi)|X_t^\pi = x),$$

where X^π is the wealth process and $\mathcal{A}(t, x)$ the set of admissible portfolios when the wealth is equal to x at the time t.

We establish the existence of a solution to the optimization problem. We associate with the optimal strategy a unique equivalent martingale measure. In a way, heading a utility function endows the investor with an "optimal" equivalent martingale measure which is identified with the Davis' probability (1997).

We give a characterization of the optimal portfolio by means of the value function and the equivalent martingale measure defined by the utility function. By the mean of this characterization, we can obtain explicit formulae for the optimal wealth, the value function and the optimal portfolio in the special cases where the utility is logarithmic or is a power function.

Furthermore, the characterization of the optimal policy allows to find bounds for prices of a European contingent claim. We show that the use of a utility function is an efficient way to reduce the range of viable prices.

Finally, we compare the value functions in continuous and discontinuous modelling. We prove that in the presence of jumps in the risky asset the investor's welfare is reduced, meaning that the value function is decreased.

The paper is organized as follows. The model is described in Section 2. The main result, which establishes the existence and the uniqueness of the optimal portfolio, is given in Section 3. Section 4 is devoted to the study of examples, and Section 5 analyzes the link between the optimization problem and the Davis' probability. Section 6 studies the price range reduction, and we compare the discontinuous model with a continuous one in Section 7.

Some proofs are given in Appendix A.

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