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The risk premium and the effects of risk on agents utility

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ABSTRACT

This work shows that the risk premium can be a mistaken measure of the reduction in utility caused by risk since, when different levels of wealth are considered, the relative size of the former is related to that of the latter only in some cases. The analysis indicates that this is because the size of the risk premium depends both on the size of the disutility due to risk and on the size of the marginal utility of money. Some simple economic problems where this conclusion is relevant are examined. The paper shows which inferences on agents behaviour can and cannot be drawn in these cases.

1. Introduction

The risk premium payable by a risk averse agent is the monetary amount that the agent is willing to pay in order to avoid a specific risk. This index, which was first studied by Arrow (1963) and Pratt (1964), is one the most famous and widely used measures of risk aversion. Its features are important for many economic problems and its size is relevant in order to measure both the insurance premium paid by an agent faced with a risk and the premium paid by a risky asset.

Despite its wide use in economic theory, certain aspects of the possible interpretation of the risk premium are still not sufficiently studied. As we said, the existence of a risk premium paid by an agent is due to her aversion to risk, meaning that it is due to the fact that the presence of a risk generates a reduction in her utility. Because of this causal link, it seems plausible to argue that the risk premium is a good measure of the reduction in utility due to risk. In other words, it seems plausible that if an agent suffers more for the presence of a risk, then she is willing to pay a larger amount to avoid it, and vice versa.

The possible correctness of this conjecture is important for two different reasons. On the one hand, we gain new insights into the interpretation of the risk premium. The literature on risk shows that, for small risks, information about the attitude of an agent toward risk obtained by the risk premium is sizeable, and is equivalent to that associated with the Arrow–Pratt index of risk aversion and the certainty equivalent. It is clear that the explanatory power of the risk premium would be further enlarged if it were shown that it also measures the reduction in utility caused by risk.

On the other hand, this issue is also relevant for those economic problems involving both the agent's manifestations of the disutility caused by risk and the amount the same agent is willing to pay in order to remove it. In some cases, in particular, it might be useful to know if the agent who shows the larger reduction in utility because of a specific risk is the same who will also pay more to avoid it and vice versa. Some examples of this kind of problem are discussed in the paper.

The aim of this work is to formally examine the conjecture that risk premium is a good measure of individual reduction in utility due to risk. The analysis is performed by comparing the risk premium with a different index called "utility premium" (or "generalised risk measure"), which is a direct measure of the disutility caused by risk. This index, introduced by Friedman and Savage (1948) and Stone (1970), was recently used by Eeckhoudt and Schlesinger (2006), Eeckhoudt et al. (2007) and Menegatti (2007), studying respectively risk apportionment of different orders, multivariate risk and precautionary saving.

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It will be shown that the above conjecture, although it is plausible, is mistaken. Indeed, it is possible that, in the general case, an agent does not pay a larger premium to escape a risk when she suffers a larger disutility for it (and vice versa). Only in some specific cases, information about the risk premium allows the inference of correct information about the disutility due to risk (and vice versa). The paper also provides a simple interpretation of these results and discusses some economic problems where these conclusions are relevant.

The paper proceeds as follows. Section 2 shows the general results of the paper. Section 3 gives a simple interpretation of these results. Section 4 presents some applications. Section 5 concludes.

2. Definitions and general propositions

We assume that an agent has a safe monetary wealth w and faces risk x, described by a random monetary wealth with known expected value and variance. Given x and w, the risk premium π can be defined as

$$E\left[U\left(w+x\right)\right]=U\left(w+\bar{x}-\pi\right) \tag{1}$$

where the function $U(.)$ (with $U'(.) > 0$) is the agent's utility function and $\bar{x}$ is the expected value of x. For a risk averse agent ($U''(.) < 0$) we have $\pi > 0$.

The well known approximation by Pratt (1964) showed that, for a small risk,

$$\pi=-\frac{1}{2}\frac{U''\left(w+\bar{x}\right)}{U'\left(w+\bar{x}\right)}\sigma^2 \tag{2}$$

where σ^2 is the variance of x.

The reduction in utility due to the presence of a risk ("the disutility due to risk") can be defined by the difference between the utility obtained when the risk does not exist and when it is present. This value is given by

$$\varphi=U\left(w+\bar{x}\right)-E\left[U\left(w+x\right)\right]. \tag{3}$$

For a risk averse agent ($U''(.) < 0$) we have $\varphi > 0$.

The index φ, called the utility premium (or the generalised risk measure) is a measure of risk aversion first introduced by Friedman and Savage (1948). Stone (1970) showed that, for small risks, we have

$$\varphi=-\frac{1}{2}U''\left(w+\bar{x}\right)\sigma^2. \tag{4}$$

It is important to note that a von Neumann–Morgenstern utility function is unique up to a positive affine transformation, while the utility premium is not invariant in this kind of transformation. This implies that the comparison of φ among agents can only be made under the assumption that their utility function is the same. This assumption will be introduced into all the comparisons between agents in the following analysis. On this point, see also the discussion in Section 5.

Given the above definitions, some general results can be obtained.

Proposition 1. *Consider a given utility $U(.)$ and a varying expected total wealth $y = w + \bar{x}$. The following relations apply:*
if $\pi(y)$ is decreasing in y then $\varphi(y)$ is decreasing in y but not vice versa;
if $\varphi(y)$ is increasing in y then $\pi(y)$ is increasing in y but not vice versa.

Proof. By using (1) and (3) together we get $\varphi = U(y) - U(y-\pi)$. This result implies that $\frac{d\varphi}{dy} = U'(y) - U'(y-\pi) + U'(y-\pi)\frac{d\pi}{dy}$. Since $U'' < 0$, the value of $U'(y) - U'(y-\pi)$ is negative. This, together with the assumption $U' > 0$, implies that the previous equation can be written as $\frac{d\varphi}{dy} = A(y,\pi) + B(y,\pi)\frac{d\pi}{dy}$ where $A(y,\pi) < 0$ and $B(y,\pi) > 0$. The relationship between the signs of $\frac{d\varphi}{dy}$ and $\frac{d\pi}{dy}$ shown by this equation proves the statements in the proposition. □

Implications

(1) Consider an agent facing a given risk x and having, in two cases, two different levels of safe wealth w_1 and w_2. In this framework:
  (a) In general, it is possible that the agent pays the larger risk premium when she suffers the smaller disutility, and vice versa.
  (b) If the agent is willing to pay a larger risk premium to escape a risk when her safe wealth decreases, then she also exhibits a larger disutility due to the risk.
  (c) If the agent exhibits a larger disutility due to a risk when her safe wealth increases, then she is also willing to pay a larger risk premium to escape the risk.

  The same conclusions are correct if we consider two agents who face the same risk x and have the same utility function $U(.)$ but different levels of safe wealth w_1 and w_2.

(2) Consider an agent facing different risks x_1 and x_2 (exhibiting different expected values and variances). It is possible that the agent pays the larger risk premium for the risk generating the smaller disutility.