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Excess demand functions with incomplete markets—a global result

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Abstract

The purpose of this paper is to give a global characterization of excess demand functions in a two-period exchange economy with incomplete real asset markets. We show that continuity, homogeneity and Walras' law characterize the aggregate excess demand functions on any compact price set which maintains the dimension of the budget set.

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1. Introduction

Sonnenschein [12], Mantel [8], and Debreu [3] proved that the aggregate excess demand function in the Arrow–Debreu exchange economy is characterized on any compact set of prices by continuity, homogeneity, and Walras' law. This research has been extended to incomplete market models with various asset structures. Similar results have been obtained by Bottazzi and Hens [1], Gottardi and Hens [4], Chiappori and Ekeland [2] and Gottardi and Mas-Colell [5]. However, these results are local; the characterization of the aggregate excess demand is given in a neighborhood of (or at) one price vector. The question naturally arises as to whether

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these properties of the aggregate excess demand function in incomplete markets are global in the sense of Sonnenschein–Mantel–Debreu. This question is the subject of the paper.[1]

We focus on an exchange economy with incomplete real asset markets. Our result is therefore an extension of Bottazzi and Hens [1]. In incomplete market economies, demand functions may be discontinuous at critical prices where the dimension of the budget set drops. It is shown that continuity, homogeneity, and Walras' law characterize the aggregate excess demand function on any compact set of prices over which the dimension of the budget set is constant.

While the set of critical prices is clearly negligible, a drawback of our technique is that the number of consumers needed for the characterization is not given by an explicit formula, although it is determined by the number of the commodities, the number of the states and the compact price set on which a candidate excess demand function is defined, and is independent of the particular candidate function. On the other hand, relying on the technique in this paper, the required number of consumers exceed the number of commodities, which is sufficient if the market is complete. Whether this is a general feature of incomplete markets would be an interesting question beyond our scope.

It should be stressed that previous methods used by Bottazzi and Hens [1] and Debreu [3] cannot be applied directly to our global problem. Such techniques fail to construct the monotone preference orderings. Overcoming this difficulty is the main technical contribution of the paper. This point is explained in Section 3.

Section 2 gives our main theorem and presents the result on the global characterization of aggregate excess demand as a corollary. Section 3 explains the previously mentioned point, which is the core of the proof of the theorem. The formal proof is given in Section 4.

2. Main theorem and result

It is sufficient to state our main theorem rather abruptly; we refer to Mas-Colell [9] for background. Let $G^k(R^n)$ denote the set of k-dimensional vector subspaces of R^n. Here $G^k(R^n)$ is a compact smooth manifold called the Grassmann manifold. Let $G^k_{++}(R^n) = \{L \in G^k(R^n) \mid L \cap R^n_+ = 0\}$.[2] For $e \in R^n$ and $L \in G^k(R^n)$, let $proj_L(e)$ denote the projection of e onto L. An *economy* consists of consumers, finite in number, where each consumer is represented by a pair $(\precsim, \omega)$, $\precsim$ a strictly convex, monotone, continuous, complete preference ordering on the consumption set R^n_+ and ω an endowment vector in R^n_+.

[1] See Hens [6] for a global characterization in a single commodity model.

[2] We write the origin of R^n as 0. This should not be confused with $0 \in R$. We also write a single element set $\{x\} \subset R^n$ simply as x.