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Non-equivalence of uniqueness of equilibria in complete and in incomplete market models*

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Summary

This paper contains two examples to prove the non-equivalence of uniqueness of competitive equilibria in complete and in incomplete market models. They show that even if an equilibrium is unique for complete markets, this property can be violated for incomplete markets, and vice versa.

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1. Introduction

The introduction of incomplete markets to the general equilibrium (Arrow–Debreu) model was a first benchmark to build up a sound foundation for the theory of finance. Since the general equilibrium model with complete markets has been vastly analysed, there have been various attempts to transfer results from the complete market model to the incomplete case (see Hens (1998) for a recent survey).

Uniqueness of the competitive market equilibrium increases the predictive power of a model in empirical applications. In addition, uniqueness is helpful for a meaningful comparative statics analysis, for example, in computable equilibrium models (cf. Shoven and Whalley (1992)).

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Arrow and Hahn (1971: chap. 9) and Mas-Colell (1985: chap. 5) found various conditions under which the uniqueness of a competitive equilibrium is guaranteed in complete market models. However, only recently has this topic been analysed for incomplete markets (cf. Bettzüge (1998) and Detemple and Gottardi (1998)). Hens, Schmedders and Voß (1999) summarize current results on uniqueness in the simplest version of incomplete market models.† In particular, they provide an example of an economy that has multiple equilibria even though consumers have Cobb–Douglas utility functions. As a by-product this example proves that even though sufficient assumptions for uniqueness in the Arrow–Debreu model are satisfied, uniqueness may be violated for incomplete markets. However, Cobb–Douglas utility functions turn out to be not the easiest example for that argument.

In this paper we give a much *simpler* example showing that we obtain multiple equilibria in incomplete markets, although the equilibrium for complete markets is unique. The economy in the example satisfies *all* the standard assumptions of incomplete market models.‡ Moreover, we give a second example in the reverse direction: The characteristics are chosen such that for incomplete markets there is a unique equilibrium, but for the corresponding complete market case there exist multiple ones. The examples show that the market structure is decisive for uniqueness, respectively, multiplicity of equilibria: the change between multiplicity and uniqueness of equilibria is only caused by different market structures, whereas *all other* characteristics remain constant. Both examples are based on Kehoe's (1991: p. 2066) well-established example for multiple equilibria in the Arrow–Debreu model.

In the examples we show that the number of equilibria is robust with respect to some small perturbations of the characteristics, including the asset structure.§

The paper is organized as follows: Section 2 describes the general setting. In Sections 3 and 4 we present the examples. We show that the incomplete asset structures can be changed a little without changing the number of the equilibria, that is, the examples are not exceptional. Section 5 concludes the paper.

† The simplest model involves a pure exchange economy with two periods, a single consumption good, and real assets.

‡ In Hens, Schmedders and Voß (1999), however, there exists no desirable portfolio that guarantees the non-satiation of asset demand for all no-arbitrage asset prices.

§ In the first example, we have to restrict the direction of perturbation of the asset structure because the asset matrix is element of the boundary of all permitted asset matrices.