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Inefficiency of equilibria with incomplete markets

Ryo Nagata*

School of Political Science and Economics, Waseda University, 1-6-1 Nishiwaseda, Shinjuku-ku, Tokyo 169-8050, Japan

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Abstract

As is well known, equilibria with incomplete markets are generically Pareto inefficient. In this paper, we demonstrate the leading role of a budget constraint in the occurrence of Pareto inefficiency of equilibria with incomplete markets. Specifically, on the basis of the classical two-period one-good pure exchange model we prove that so long as a budget constraint is met for all agents, equilibria with incomplete markets are generically Pareto inefficient in initial endowments and utility functions regardless of the optimization behavior of each agent. All we require of utility functions is a very weak hypothesis called current monotonicity. A simple unified method applicable to both a real asset case and a nominal asset case is presented, so that our claim is proved in both cases.

JEL classification: C 60; D 52

Keywords: Incomplete market; Current monotonicity; Budget optimum

1. Introduction

It is well known that with incomplete asset markets, the equilibrium allocations need not be Pareto optimal, as Hart (1975) first suggested. Since Hart's work, many efforts have been made to investigate if a competitive outcome with incomplete markets is constrained optimal in some sense. The idea of constrained optimality itself has been formally shaped

* Tel.: +81 3 5286 1227; fax: +81 3 3204 8957.
E-mail address: rnagata@waseda.jp.

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through the works of Diamond (1967), Stiglitz (1982) and Geanakoplos and Polemarchakis (1986) into the following notion. An allocation with incomplete asset markets is constrained optimal (or constrained Pareto optimal) if and only if it is not Pareto dominated by any other allocations that can be obtained by a social planner who can control only existing asset markets. With respect to this notion, however, it has been shown that except for a restricted case (i.e., the one-good two-period case) competitive equilibria with incomplete asset markets are generically constrained suboptimal, which substantially means that they are typically not constrained optimal (see esp. Geanakoplos and Polemarchakis, 1986). The generic constrained suboptimality has subsequently been investigated in variously elaborated contexts in the literature (see Geanakoplos et al., 1990; Werner, 1991; Kajii, 1994; Cass and Citanna, 1998).

In contrast to many works concerning a modified optimality concept of equilibria with incomplete markets, there is only a small amount of literature that deals with Pareto inefficiency of equilibria with incomplete markets itself (Magill and Quinzii, 1996; Villanacci et al., 2002). To this more fundamental issue, this literature has shown that such allocations are generically Pareto inefficient with respect to the agents' initial endowments with the assumption of concavity of utility functions of agents.

In this paper, we consider the latter problem from a different viewpoint. Our concern is what determines Pareto inefficiency of equilibria with incomplete markets. With respect to this issue, we show that such inefficiency is not dependent on the optimization behavior of agents. More specifically, such inefficiency happens to those equilibria not because an objective equilibrium (market clearance) is accompanied by a specific subjective equilibrium (optimization) of each agent but because it is accompanied by a budget constraint of each agent. Alternatively put, so long as a budget constraint is met for all agents, those equilibria are generically inefficient regardless of each agent's optimization behavior based on its own consumption. Thus it may be safely said, though in a generic sense, that once the agents participate in incomplete markets, they are kept away from Pareto optimal allocations before they declare their demand.

In order to prove our claim effectively, we must bear some aspects in mind. One is to adopt less specified utility functions for agents. To this end, we consider a very weak monotonicity called current monotonicity as an assumption. This only requires monotonicity of utility with respect to consumption at present. We do not set other assumptions, particularly any concavity, on utility functions as other authors do. The type of assets is also our concern. As is well known, assets are conceptually classified into two groups, that is, real assets and nominal assets. A real asset promises to deliver a bundle of goods at each state in the future, whereas a nominal asset promises to deliver a given stream of units of account across the states. It is noteworthy about these two kinds of assets that the structure of the set of equilibrium allocations is very different between them. It is shown, though on the basis of the concavity assumption, that in a real asset model the equilibrium set is generically finite Duffie and Shafer (1985) whereas in a nominal asset model there generically exists real indeterminacy of equilibria (Cass, 1984, 1985; Werner, 1985). The real indeterminacy of equilibria indicates that the set of equilibrium allocations constitutes a continuum. Moreover, it has been shown that the continuum set generically contains a definite dimensional manifold and that the dimension of the manifold is $S-1$ or $S-J$ according as to whether the nominal asset prices are taken to be endogenous or exogenous (Geanakoplos