Discreteness of equilibria in incomplete markets with a continuum of states

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Abstract

We discuss the issue, raised by Mas-Colell [Mas-Colell, 1991. Indeterminacy in incomplete markets economies. Economic Theory 1 (1), 45–61] whether local uniqueness (relative to the L^∞ topology) may be a generic property of equilibria in incomplete markets economies with a continuum of states. © 2000 Elsevier Science S.A. All rights reserved.

1. Introduction

The analysis of equilibria in incomplete markets economies with a continuum of states is still an important, open and difficult subject. The known results on existence of equilibrium use either a strong assumption on non-negativity of ex-post endowments — i.e., endowments plus real returns (see Hellwig, 1996; Mas-Colell and Zame, 1996; Monteiro, 1996) — or a modification of the equilibrium concept to allow for default subject to penalties (see Araujo et al., 1996, 1997). The cardinality of the equilibrium set was addressed by Mas-Colell (1991).

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The generic finiteness of the equilibria does not extend from the complete markets case to incomplete market and the difficulty has to do with the nature of the continuum states set (in contrast with the well-known difficulties, limited to nominal assets, faced when the states set if finite). Mas-Colell (1991) gave a robust counterexample of a continuum of states incomplete market economy with uncountably many equilibria.

Mas-Colell's goal was to examine only the cardinality issue, but in his own words, ''It is certainly true that for comparative statics purposes, local uniqueness is the key property.'' The choice of topology is crucial for discreteness of equilibria: in the case of the (separable) L^1 topology a continuum of equilibria is a non-discrete set, but in the case of the non-separable L^∞ topology discreteness is compatible with a continuum of distinct equilibria. In Mas-Colell's challenging words, ''It could be asked if local uniqueness (relative to the L^∞ topology) may still in some sense be a generic property. This is a difficult question and we have no answer to offer.''

Our purpose is precisely to obtain results on discreteness of equilibria under the L^∞ topology. In the case where there are no trades in assets markets, the pure spot market equilibria of the incomplete market economy are shown to be discrete, generically, on endowments and utilities. Moreover, if utilities are state-independent, discreteness is generic on endowments. Our result on the no-assets case is interesting since this case has been studied previously in the literature, by Mas-Colell (1991) and Davila (1998) having been pointed out by Mas-Colell (1991) that ''...in what concerns the indeterminacy question the trivial no assets case is not trivial at all, but it represents perfectly well what happens in the general incomplete situation.''

When assets are traded, we show first that, generically on endowment functions and assets returns matrices, ex-post spot endowments are regular, for almost every state. In particular, this shows that the regularity assumption in the work of Mas-Colell and Monteiro (1996) on existence of equilibrium is actually, generic. Secondly, we establish the discreteness of equilibria with asset trades for an open dense set of endowments, provided that utility is state-independent and the set of non-regular spot endowments translated by the span of the assets return matrices has an empty interior.

2. The model and definitions

Consider a two-period economy $\mathscr{E} = ((w^i, u^i)_{i=1}^{I}, A)$ with a finite number I of agents. There is no consumption in the first period. There is a continuum $\mathbb{S}$ of states of nature that may occur in the second period affecting endowments, assets returns and possibly also preferences over G physical goods. We endow $\mathbb{S}$ with a σ-algebra $\mathscr{B}$ and a non-atomic probability measure μ. Preferences are assumed to be additively separable over states and the instantaneous utility functions belongs