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Incomplete Markets and Volatility

Laurent E. Calvet[1]

Department of Economics, Littauer Center, Harvard University, Cambridge, Massachusetts 02138-3001
laurent_calvet@harvard.edu

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This paper shows that the precautionary motive, combined with asset incompleteness, is a major source of volatility and indeterminacy in financial markets. Price fluctuations originate from agents' efforts to insure themselves through time by borrowing and lending instead of shifting income across states of nature by trading risky assets. A *high* interest rate at a future date reduces the potential for future consumption smoothing via borrowing, which leads to a strong precautionary motive and a *low* interest rate in the current period. The negative feedback between future and current rates generates fluctuations. This logic is developed in SPEC, a CARA-normal exchange economy with many periods and endogenous interest rates. When there is an intermediate level of market incompleteness and sufficient investor impatience, fluctuations in the real interest rate can be large, even though the aggregate endowment is constant. SPEC has a unique equilibrium under a finite horizon; on the other hand, with a finite number of infinitely lived agents, there exists a robust continuum of equilibria that are neither bubbles nor sunspots. *Journal of Economic Literature*. Classification Numbers: C61, D52, D58, G11, G12.

Key Words: CARA-normal; endogenous fluctuations; exchange economy; financial structure; general equilibrium; incomplete markets; indeterminacy; precautionary motive; volatility.

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1. INTRODUCTION

Does market incompleteness cause price fluctuations in financial markets? We answer this question affirmatively by introducing SPEC,[2] a special general equilibrium model in which the dynamic path of equilibrium prices can be explicitly calculated for any sequence of asset structures. Intermediate levels of market incompleteness give rise to large temporal fluctuations in macro variables, even when exogenous aggregates are constant.

SPEC is a special case of a many period CARA-normal exchange economy, with the special feature that interest rates are endogenous. Because of its simplicity, the model allows us to analyze the interaction between asset span and intertemporal volatility. In a perfectly competitive economy, we consider finitely many agents with identical CARA utilities, symmetric information, and heterogeneous time-dependent endowments. The financial structure and aggregate endowment are deterministic in each period, but can vary with time. In equilibrium, individual consumption is random, but the macro variables are deterministic and non-stationary. For an intermediate level of market incompleteness and sufficient investor impatience, fluctuations in the real interest rate can be large, even though the aggregate endowment is constant.

Investors can insure themselves *across states of nature* by buying and selling assets, and *across time* by borrowing and lending. In SPEC, the first form of insurance does not cause market fluctuations because agents have identical CARA preferences and there is no aggregate risk. On the other hand, self-insurance by borrowing and lending can generate large endogenous movements in equilibrium prices. When an agent anticipates a *high* interest rate in the future, she realizes that it will be costly to borrow in future states where she will be poor, leading to a high precautionary motive and a *low* interest rate in the current period. The opposite movement in future and current interest rates generates fluctuations. When markets are complete, agents can fully insure themselves by transferring wealth across states of nature via risky assets; and thus the precautionary motive and market fluctuations disappear. Conversely, in the absence of risky assets, investors are exposed to large uninsurable risks and have a strong precautionary motive in every period; interest rates are then low and fluctuate in a narrow range along the equilibrium path.

The equilibrium calculation builds on a new result in consumption-portfolio theory. Given an exogenous stream of spot prices, a trader's maximization problem is solved in the CARA-normal case under both finite and infinite horizons. In contrast, previous authors have solved the infinite

[2] SPEC is an acronym for Savings, Precaution, and Endogenous Cycles.